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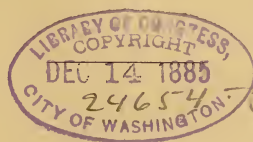
PHYSICAL LABORATORY NOTES.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

BY

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PREFACE.

THESE notes are a portion of the directions which have been prepared from time to time during the past ten years as an aid in class instruction at the Rogers Laboratory of Physics of the Massachusetts Institute of Technology. They are now printed (but not published) by the Institute for the convenience of its students. About one-half of the notes up to page 36 are supplementary to directions given in Pickering's "Physical Manipulation," vol. i., but the greater portion of the remainder is entirely independent of any text-book.

The treatment of the subjects of Thermometry and Heat-Measurements is original. The experiments there described have been found, by experience with classes, to be satisfactory both as to accuracy of results and value for training in careful manipulation. Few, if any, existing text-books give an exact statement of the proper use of the mercurial thermometer with an accuracy of 0.1° to 0.01° C. below 100° C. In view of this fact, and of the necessity of this knowledge in many branches of professional work, it is thought that the section on Thermometry will be of special interest. The omissions in regard to the use of very sensitive thermometers and to measurements of very high temperature are intentional, these points being considered beyond the scope of this class of laboratory instruction. The notes on Electrical Measurements are designed rather for students who are beginning a somewhat elaborate course in that subject than for general students. The notes on Photography are by Mr. W. H. Pickering, Instructor in Physics at the Institute.

It will be seen that, throughout the course, the work is quantitative. In connection with the later portions, and with the more advanced work which follows, attention is directed to the study of all sources of error and to the proper adjustment of the relative precision in the component parts of complex measurements.

SILAS W. HOLMAN.

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PHYSICAL LABORATORY NOTES.

VERNIERS.

THE object of a vernier is to read correctly and with facility the position which a reference mark occupies in regard to a graduated scale, *e.g.* the position of the mark 0, Fig. 1, beside the scale *AB*. By eye estimation, this might be read as 11.6 plus a certain fraction of *ac*, which might be estimated as about 0.3 of *ac*; so that the reading would be 11.63. The vernier is designed to give this fraction with greater certainty, and without the estimation by the eye. For this purpose there is laid off upon the sliding part *CF*, Fig. 2, carrying the reference mark, a length *de* equal to a certain number *m* of the smallest scale divisions; and this length *de* is divided into one more (or one less) parts than *m*. In the second figure, *CF* represents the first vernier of the series used in the experiment, and *m* = 9, *de* = 10. Thus each division of the vernier is $\frac{m}{de} = \frac{9}{10}$ of a scale division. Thus, if the

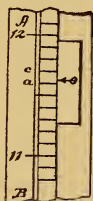


FIG. 1.

line 0, *i.e.* the zero-point of the vernier, be made to coincide with any line, *e.g.* 8, of the scale, the next vernier division, 1, will fall below *G*, the next scale division, by $\frac{1}{10}$ of the distance between 8 and *G*. Consequently if the vernier be moved until the line 1 coincides with the line *G*, the zero-point will have been advanced 0.1 of *G8* beyond 8, but *G8* = 0.1 of the scale unit (*i.e.* of the space between 8 and 9 on the figure); hence the vernier has been moved 0.01 of a scale unit past *D*, and the space (corresponding to *ao* in Fig. 1) which we desire to know is 0.01 of a unit, the whole reading thus being, in the supposed case, 8.01. If the line 2 were made to coincide with *F*, the reading would be 8.02, and so on. In the same way with any other vernier, find how many divisions *m* upon the scale are comprised in the whole length of the vernier, and into how many parts *n* the vernier is divided. Then the vernier division is one $\frac{m}{n}$ -th of a scale-

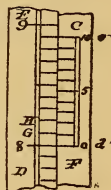


FIG. 2.

division, and the difference in length between a vernier and a scale division is $\frac{1}{n}$ -th of a scale division. Find, next, into how many parts *p* a scale unit is divided. Then the vernier division will be shorter (or longer) than a scale division by $\frac{1}{n} \times \frac{1}{p}$ of a scale unit. Hence, if the first division of the vernier coincides with a scale division, the space *ao* is $\left(\frac{1}{n} \times \frac{1}{p}\right)$; if the second line coincides, $ao = 2 \left(\frac{1}{n} \times \frac{1}{p}\right)$, and so on, as shown numerically in the example just given.

In the first vernier of the series, *m* = 9, *n* = 10, and *p* = 10; thus $\frac{1}{n} \times \frac{1}{p} = 0.01$, and the vernier reads to 0.01 of a scale unit, as already shown. In the third vernier, *m* = 24, *n* = 25, *p* = 20, and the vernier thus reads to $\frac{1}{25} \times \frac{1}{20} = \frac{1}{500} = 0.002$. To read a vernier, having already determined the fraction to which it reads, take first the whole of the scale reading up to the line *a*, Fig. 1, next preceding the

zero-point of the vernier; then follow along up the vernier to the line which is just opposite one on the scale. Multiply the number of this line from the zero by the fraction to which the vernier reads, and add this to the scale reading. The vernier is, however, usually so numbered as to render this multiplication unnecessary, and the following is a somewhat simpler method of reading, which may be adopted when the principle of the instrument is understood. Consider the vernier as simply a magnified and subdivided scale division. Thus, in the first vernier, the whole length of the vernier would be a representation of 0.1 of a scale unit, *i.e.* of one of the smallest scale divisions magnified and subdivided into ten parts; so that the vernier reads to 0.01 of a unit. In reading, take first the whole scale reading to the line next preceding the zero of the vernier; then append to this decimal place the next as read from the vernier (or add the vernier reading, if other than a decimal), by noting the line coinciding with one on the scale. The numbering on the vernier should show at a glance the fraction to which it reads. When no line of the vernier is exactly in coincidence with one on the scale, the line nearest to coincidence is read; or the precision may be increased by estimation.

Become familiar, by repeated setting, with each of the verniers in the block used. Then set the verniers to the following readings: 1st, 8.03; 2d, 29.9; 3d, 30.866; 4th, $4^{\circ} 10'$; 5th, $0^{\circ} 17'$; 6th, $2^{\circ} 58' 30''$; 7th, $48^{\circ} 52'$. Then allow an instructor to see that the settings are correct, and to set the verniers to be read.

INSERTION OF CROSS HAIRS.

Read "Physical Manipulation," i. pp. 29-31.

Use thin card-board. Draw two fine lines, intersecting at *C*, at right angles to each other. With *C* as a centre, describe an outer circle of a diameter slightly



FIG. 3.

smaller than the inside diameter of the tube, and an inner circle a little larger than the aperture of the diaphragm within the tube. Cut out this ring, and, by placing it in the tube in the position it will occupy when in use, see that it fits without binding. Pin the ring down to the papered side of the block at points *A* and *B*. At the point *D*, on the extension of one of the lines intersecting at *C*, insert a pin. Draw out two silk fibres, and to the ends of each attach small bits of paper, so that they may be easily seen and picked up. Wind one end of the fibre about the base of the pin at *D*, and stretch it across the ring over one of the intersecting lines. Holding it in this position, secure the fibre at *L* by a piece of mucilaged paper. Then with strips of mucilaged paper secure the fibre in its place upon the ring at *F* and *H*. Cut the fibre close to the ring, and, moving pin *B* to *K*, proceed in the same way to attach the cross fibre. The projecting ends of the paper strips should be allowed to remain. They serve, by their friction upon the inner surface of the tube, to hold the ring in place.

ECCENTRICITY OF CIRCLES.

In the use of any such instrument as the theodolite, compass, goniometer, etc., where angles are to be read from a graduated circle by means of verniers or other devices, and when the full accuracy of which the circle is capable is desired, correction must always be made for the eccentricity of the mounting, unless this be proved so small as to introduce only a negligible error. The

eccentricity arises from the difficulty of so constructing the instrument that the axis of rotation of the plate to which the verniers are attached shall pass exactly through the centre of the graduated circle. Thus in the figure let A and B represent the zero-points of a pair of verniers intended to be at opposite ends of a diameter of the circle; but suppose AB , by imperfect construction, to pass on one side of the centre of graduation C . Suppose also that C' is the axis about which rotates the plate to which A and B are rigidly attached. This line AB may or may not pass through C' ; in general it does not. If C' coincide with C , there will be no eccentricity, and one vernier will suffice. The use of the second vernier is simply to eliminate the error due to almost unavoidable eccentricity. The nature of this error can be seen by an inspection of the figure, and will be seen to depend on the distance of C' from C , and of the line AB from C' , the former causing a periodic, the latter a constant error.

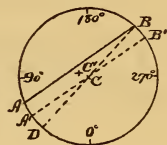


FIG. 4.

Suppose the verniers are in any such position as at A and B in the figure, and call A and B their respective observed readings. The line $A'B'$ is drawn through the centre of graduation C , and parallel to AB . Then A' and B' are the true readings corresponding to this setting of the instrument, for these are the readings which A and B would have if AB passed through and revolved about C , the proper centre. Now it is necessary to find from A and B the true reading of one or the other vernier, say the reading A' . From B draw through C the straight line BCD . Remembering that with the numbering of the circle as shown in the figure, the reading B is the length of the arc $0, A, 90, 180, B$, it will be seen that $B - 180^\circ = 0D$. Also from the figure, $A'D = A'A$. Therefore $B - 180^\circ$ is as much smaller than the true reading A' as the observed reading A is greater than A' ; or *vice versa* if AB happens to be on the other side of $A'B'$. Thus in any case, until B passes the 0° point, that is, so long as B is numerically greater than A , the corrected reading of vernier A will be the mean of $B - 180^\circ$ and A , and may therefore be found either by the expression $A' = \frac{1}{2}(A + B - 180^\circ)$, or, by the rule, subtract 180° from the reading of B , and find the mean between this difference and A .

When B is numerically less than A , that is, after B has revolved past the zero-point, it becomes necessary to add 180° to B instead of subtracting, as will be readily seen from a figure drawn to suit this case. Insert the figure and demonstration for this case in the note-book.

With the circle used in this experiment, start with A at about 0° , and read both A and B . Turn A to about 20° , and read both verniers as before. Continue at intervals of about 20° throughout the whole circumference. Afterward compute the true reading for A in each of these positions.

To study the nature of this error, compute the error of A at each observed reading, and construct a curve with readings of A as abscissas, and errors as ordinates. If AB pass through C' , this curve will be half above and half below the axis. In general this is not the case. State in the note-book what would be the form of the curve, and its position in regard to the axis: first, when AB passes through C' , but C and C' do not coincide; second, when AB does not pass through C' , but C' and C are coincident.

Circles are frequently numbered from one zero-point both ways to 180° . An inspection of the figure will show that for this case $180^\circ - B$ will equal $0D$. Hence $\frac{1}{2}[A + (180^\circ - B)]$ gives A' . And from this it is also evident that when, as in compass circles, the numbering is up to 90° in each of the four quadrants, running both ways from two zero-points which are at opposite ends of a diameter, it is only necessary to take the mean of A and B ; that is, $A' = \frac{1}{2}(A + B)$.

SPIRIT LEVEL.

The spirit-level consists of a glass tube AB , of which the upper surface is so curved that a longitudinal section would be the arc of a circle of several feet radius, the sensitiveness of the level being greater as the radius of curvature is greater. The tube is nearly filled with pure alcohol, or a mixture of alcohol and a small proportion of sulphuric ether,

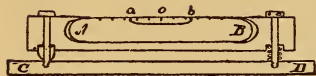


FIG. 5.

the latter being a more mobile liquid, rendering the bubble more prompt and sure in action. On account of its greater expansibility, ether cannot be used alone unless the tube is provided with a suitable expansion chamber at the end containing a bubble of considerable size.

The tendency of a free liquid surface to become a plane perpendicular to the line of the earth's attraction causes the bubble always to approach that point of the arc at which such a plane would be tangent to it. The direction of the plumb line would be always perpendicular to this plane, which would of course deviate from the horizontal just as much, and by the same causes, as the plumb line from the vertical, — often a measurable amount.

The tube when mounted is usually provided at one end with an adjusting screw, by which the lower surface or base of the level may be rendered parallel to the tangent plane through the centre division o of the tube; that is, so that when CD is placed on a horizontal surface, the ends of the bubble may be made equidistant from the centre line.

To make the adjustment, place the level on any nearly horizontal and quite smooth surface, as a plate of ground glass, and read the position of each end of the bubble, calling these a and b respectively. Note carefully, or mark on the plate, the positions of the ends of the level, and then reverse it as exactly as possible. Call a' and b' the readings in the new position. In what follows it is assumed that the numbering upon the level is from the centre o in each direction, and that the readings a, a' , etc., and b, b' , etc., are respectively in the directions oA and oB , the ends A and B being permanently lettered on the instrument.

A consideration of the conditions will show that if $b' = a$ and $a' = b$ the level is in proper adjustment, otherwise the screw at one end must be turned in the proper direction, without moving the whole instrument, until the bubble is made to take a position at either end about half way between the positions b' and a or a' and b . This operation must be repeated until the same readings ($b' = a$ and $a' = b$) are obtained upon reversal. If, however, the plate is not approximately level (the degree of approximation depending on the sensitiveness of the level), it must first be made so by the use of the method described in the following paragraph. Also, if the level is so far out of adjustment as to cause one end of the bubble to disappear beneath the metal at one end of the tube, the rule in regard to halving the differences of positions between b' and a , etc., is not to be followed, since the curvature of the glass towards the end of the level is greater than in the middle, and consequently, the adjustment would be accomplished by a motion of the adjusting screw which would move the visible end of the bubble much less than half the distance between b' and a . In any such case two or three approximate adjustments will be requisite.

Levels attached to instruments are, for convenience, often adjusted and permanently fixed in position, but must usually be adjusted at each time of use. Adjustment is not, however, always essential, for if an unadjusted level be placed upon a horizontal plane surface, the reading of the end of the bubble will

remain unchanged, whatever the position of the level upon the surface. Hence, to level with an unadjusted level, it is only necessary to move the surface (by means of the levelling screws provided) until, on being turned into any position on the surface, the level shows no change of reading. Of course a plane surface will always be level when two lines at right angles upon it are level; and where, as is usual, a surface has three levelling screws, the quickest manner of levelling it is to level one line parallel to the line joining two of the screws, and another line at right angles to this. Adjust the level, level the glass plate, and test the levelling of the plate by means of the level when thrown slightly out of adjustment. Record all the numerical readings taken in the adjustment of the level as described, and also by figures demonstrate the method of adjusting the level by reversion on an approximately level surface, and the method of levelling with an instrument out of adjustment.

On engineering and astronomical instruments it is sometimes necessary, either for the determination of errors or for the accurate measurement of small vertical angles, to know with precision the angular displacement of the level corresponding to the motion of the bubble over one division of the scale on the tube. This quantity may be measured as follows, and in sensitive levels may be as small as a few seconds of arc.

Upon a firm table FG is placed a rigid bar AB supported at the end A by two projecting points which rest on a plate of wood or metal, and carrying at the end B a micrometer screw whose point rests on a flat plate C , of metal or glass. The distance from the point of the micrometer to the line joining the points at A should be measured. The degree of precision of this measurement should depend on the precision with which the bubble can be twice set at the same point in the tube; and as this will seldom exceed or equal the hundredth part of a division, the measurements of AC , and by the micrometer, should be made to about 0.1 per cent. The level is placed on the bar, preferably near one end. The micrometer is turned until the end of the bubble is at any desired mark (say 5), and the readings of both micrometer and level recorded. This is repeated several times at the same mark, with the precaution of always moving the bubble up to the mark from the same side to eliminate differences of friction within the tube. Make a number of settings of the same end of the bubble at the mark just used, but by moving the end up to the mark in the opposite direction from that before used. Take the mean of this series of micrometer readings, and find the difference between this mean and the preceding one for the same division. Compute the average deviation of each series, and determine if possible from these whether the difference between the means is purely accidental, or is probably due to tardy action of the level.

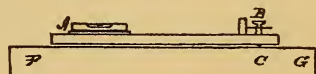


Fig. 6.

Make a similar series of settings from one side only for four successive divisions of the level, recording the numbers of the marks used, and whether towards A or B . Compute the means for each division. The difference between the means for successive divisions will give the difference of micrometer reading corresponding to the interval between these two particular divisions of the level. From this compute the angle corresponding to each observed pair of successive lines. These will or will not be equal according to the uniformity of curvature of the surface of the tube. In the calculations note that the difference in micrometer readings divided by the measured distance AB , expressed in the same unit, gives the tangent of the angle through which the level is turned; and that, as this angle is very small, its tangent coincides closely with its arc. Also, in any circle, the arc whose length is equal to the radius measures $206265''$.

ESTIMATION OF TIME OF TRANSIT.

A weight having a large moment of inertia is suspended before a bright window by a fine steel (piano) wire of such diameter and length that the time of a single torsional vibration is about half a minute. To this weight is secured a horizontal radial rod about half a meter long, carrying at its outer end, and at right angles to it, a vertical disk 2 or 3 decimeters in diameter, which is pierced near its centre by a small hole. The torsional vibrations of this apparatus, when once started by a slight twist, will decrease in amplitude so slowly as to continue sufficiently large for observation during a quarter of an hour. A telescope with a vertical wire or with cross wires in its eye-piece, is placed at a distance of about 50 feet from the disk. The observation will correspond to the ordinary "eye and ear method" of observing the time of transit of a star as employed in astronomical observation.

First adjust carefully the position of the cross wires or of the eye-lens so that the cross wires shall appear perfectly distinct. Then focus the telescope carefully on the disk, so that the spot of light and the cross wires of the telescope are both clearly seen at the same time. Point the telescope to about the middle of the path of the artificial star. The observation to be made is of the times of successive transit of the spot of light as it moves in the *same* direction across the vertical wire of the telescope. Start the metronome with its sliding weight set at 60, *i.e.*, so that it shall beat seconds. Count the seconds continuously, observing through the telescope at the same time the position of the star at each stroke or second. The appearance will be somewhat as the sketch. Let the circle represent the field of view of the telescope, and the lines the cross-wires. The dots show the successive positions of the star at (say) 3, 4, 5, 6, 7, 8, etc., seconds. Then the star crossed the vertical wire at some time between the 6th and 7th seconds; and as the wire is at about 0.4 of the way from the



FIG. 7.

position at the 6th to that of the 7th second, and as the star moves nearly uniformly at that part of its path, the transit must have taken place at 6.4 seconds. The counting is to be made continuously, and this having been done, the star will be found again approaching the wire in the same direction as before (the time of transit in reverse direction is not wanted) at (say) the 43rd second; and the position at 43 and 44 seconds having been mentally noted, as before, it would be estimated that the transit occurred at 43.8 seconds. These observations are to be made continuously, recording at each transit the corresponding second and tenth of a second, until the time becomes 100 to 200 seconds, when a new series may be begun. In the counting, it will be found easier to count up to 50 and then drop the tens' place of figures, going on with 1, 2, 3, etc., and putting in afterward the necessary additional tens.

Difficulty is frequently experienced in the first attempt at the observations, from the necessity of making the estimation, record, and count, at the same time. It is therefore sometimes best, in the first few observations, not to attempt to keep up the counting beyond the first transit, but to make several disconnected observations until a little practice in the estimation is obtained.

Every observation made should be recorded in the note-book.

Since the rate of vibration of the pendulum is constant, the intervals between the times of successive transits should be equal. Find, therefore, the first differences and their average deviation from their mean. This gives a measure of the precision of the estimation.

METHOD OF WEIGHING.

The student unfamiliar with the use of a balance of even moderate delicacy should follow carefully the following suggestions and directions, so that he may avoid injury to the balance, and become methodical in making weighings, an essential to rapid and accurate work with the balance.

The beam is supported by "stops" or bearings of one or another form, and it or the supports are movable by some mechanical device so that the beam may be made to rest on its knife-edge bearing for weighing. This motion is usually made by turning a milled head, by raising or lowering a lever, or by some device apparent on inspecting the balance. The beam must be lowered very slowly and gently upon the knife-edges, as sudden blows or jars dull and spoil the edges and their supporting surfaces, and thus greatly injure the delicacy of the balance. No weights or substances, however small, should be placed upon the pans or taken from them; nor should any operation whatever, involving a touch however light upon the balance or its case, be performed while the beam rests on the knife-edges. Weighing is a slow and delicate manipulation, and should be carefully and thoughtfully performed.

Some balances are furnished with light "stops," which may be raised and lowered beneath the pans so as to just touch the bottom of the pans when the beam is horizontal. In weighing, these stops should be raised against the pans until the beam is lowered upon the knife-edges, and should then be slowly lowered clear of the moving pans. They serve to lessen the blow upon the knife-edges, to stop the lateral swinging of the pans, which must never be allowed in weighing, and to lessen the strain upon the beam as the loads are being placed in the pans.

When about to weigh, see first that the balance is level, turning the foot screws, if necessary, until the level, plum-bob, or other means of levelling, if any is provided, shows that the balance is level. See next that the beam is in its proper place—not twisted from its normal position on the stops by previous careless usage—and that the scale-pans hang properly from their knife-edges or supports. Lower the beam carefully, lower also the pan-stops slowly, and see whether the pointer or index moves equally on each side of the middle division of the scale at its foot, keeping the balance case shut to avoid disturbance by air currents in this and in the final parts of all weighings. If it does not, the balance is out of adjustment, and must be readjusted before weighings are made. For instruction upon this point, call upon the instructor. Immediately upon the completion of every observation on the swing of the balance, before any other motion is made, the beam must be raised from the knife-edge, turning up first the pan-stops, if these are present.

The substance to be weighed is placed in the left-hand pan, the weights in the other pan, on account of the greater safety and convenience of transferring the numerous weights from the box at the right hand to the pan nearest it. This practice should be habitual, as, in addition to its convenience, it eliminates the effect of unequal balance arms where only relative weights are desired. The error from unequal balance arms may be eliminated either by the method of double weighing, or by "tareing" (Kohlrausch, *Phys. Measurements*, p. 29; Thresh, *Expt'l Physics*, p. 23), or by measuring the ratio of the two arms as described in the notes on "Adjusting and Testing a Balance." All weights, except when very heavy and intended for rough work *only*, should invariably be handled by pincers. By moisture from the hand the weight may be increased by even a mgr., and rusting will ensue which will destroy the pre-

cision of the set of weights. For small weights, from 10 mgr. down, it is more convenient to use a "rider" than small metal weights. The rider is a bent piece of wire which may be moved along the graduated balance arm and hung upon it at any desired point, or removed by means of a carrying rod which projects through the end of the balance case.

In making the weighing, follow closely this manner of using the weights. See that the rider swings wholly clear of the beam. The substance being in the left pan, and the box of weights open upon the table (not on the balance case) in front of the right pan, transfer to the middle of the pan a weight which is thought to be somewhat larger than that required, — suppose 50 gms. Lower the stops slowly — generally it is unnecessary, until the weights are nearly right, to lower completely — until the index moves to (say) the left, showing that 50 is too large. Raise stops, and return the 50 to its place in the box (never place weights anywhere but in their places in the box or in the scale-pan), and put on the next smaller, 20 gms. On lowering, index goes to right; 20 is too small. Put on the second 20 (sometimes two twenties and one ten are provided, sometimes one twenty and two tens, — the former is preferable); too much. Remove, and put on 10; too little. Put on 5; too much. Remove, and put on 2; too much. Remove, and put on 1; too little. Put on 0.5; too little. Put on 0.2; too little. Put on other 0.2; too much. Remove, and put on 0.1; too much. Remove, and put on 0.05; too little. Put on 0.02; too little. Put on other 0.02; too little. It is unnecessary to now put on 0.01, because that would make up $0.05 + 0.02 + 0.01 = 0.10$, which has already been found too much. Add the remainder by the rider, keeping the balance case closed, finding such a point for it that the index will swing, as at the outset with no load, equal distances on each side of the middle point. If, in the final weighing, the balance does not readily swing of itself, a touch with the pan-stops or a very light touch with the pincers on one pan will suffice to give the motion of four or five divisions on each side. Then proceed to count up the weights missing from their places in the box, — for each weight should have a fixed place in the box, — and add to this sum the weight corresponding to the position of the rider (suppose 0.0024 gms.). In this case the sum would be 31.7924 gms., and the figure in each decimal place should be put down as it is read off from the box. Then open the balance-case and count up the weights in the pan, beginning with the largest, and noting, as each place of figures is completed, whether it checks with the already recorded number. Then read and *remove* the rider, and remove the weights in order, beginning with the largest, counting up the weights a third time, and noting whether this checks the previous numbers. By this repetition, which takes very little time more than is required for the mere manipulation, annoying mistakes in recording weights may be wholly avoided.

On leaving the balance, always see carefully to the raising of the beam from the knife-edges, the removal of the rider from the beam, and the closing of the balance case, and of the box of weights, in which the pincers should be left.

To familiarize himself with the above procedure, the student should make the weighing to 0.1 mgr. of each of the two pieces of metal provided for the purpose, and then of the two together, as a check, recording the weights against the marks found upon the pieces.

ADJUSTING AND TESTING A BALANCE.

A delicate balance should be properly protected from dust and excessive moisture. Drying substances within the balance case are sometimes, but not always, of service. They may maintain the air within the case at so low a humidity that sensible condensation or absorption of moisture by bodies being weighed does not occur. The balance should not be exposed to unequal or irregular heating, and should stand on a table as free as possible from jarring. Where a slight vibration is unavoidable, it may be lessened by placing masses of rubber beneath the feet of the case; but, as such a support is somewhat yielding, care must be taken to see that this does not introduce consequent errors into the weighings. In what follows, the balance is supposed to be so located, and to have been properly cleaned and put together. It remains to test and adjust it, and the following notes give the main points in this process. The student should note the number of the balance to which he is assigned; and should record against the number corresponding to the descriptive section the observations and result obtained with the balance. Students unfamiliar with the use of a delicate analytical balance should perform the work indicated in the notes on "Method of Weighing" before going on with the work of these notes.

1. **Levelling.**—Level the balance by either the plumb line behind the standard, the level on the base of the standard, or the level in the case, according to which is furnished with the balance. Great precision in levelling is not essential, but firmness of the balance case, so that this adjustment, once made, shall not be disturbed during the subsequent use of the instrument, is important.

2. **Equilibrium.**—Release the beam from the stops; *i.e.*, lower it by the method provided for the purpose until the knife-edges rest upon the supporting plates. This motion and the reverse one must invariably be gentle, avoiding any sudden blows upon the knife-edges, or jarring of the beam or pans. The stops of the balance, when properly arranged, will release both sides of the beam at the same time. No changing of the load in the pans, no raising or lowering of the balance-case doors,—in short, no operation whatever on the balance should be performed, except in some cases by the experienced manipulator, while the beam rests upon the knife-edges. The pans must not be in vibration when weighings are made. When the beam is thus released, the index should swing freely and slowly equal distances on each side of the middle point of its scale. If it does not, see first that the pans are properly hung upon the beam, and then turn the adjusting nut until the index swings approximately equal distances on each side of the middle mark. A nut at one or each end of the beam, moving on a screw projecting from the end of the beam, or sometimes a small projecting piece or "flag" of metal revolving on a vertical axis at the middle of the beam, constitutes the attachment for this adjustment. If this adjustment cannot be made, but the balance, on starting to one side or the other, continues to swing in that direction with increasing velocity, tending to overturn, then it is in unstable equilibrium, and the large vertically moving adjusting nut at the middle of the beam (above or below) must be lowered until the proper equilibrium is attained. In the ordinary analytical balance, the time of a single vibration (swing from one extreme to the other, or, better, from the middle point to one extreme and return) should be from 5 to 15 seconds.

Test whether the balance is in stable equilibrium, and note the time of the single vibration. This time can be regulated to the proper one for the purpose of the balance by raising or lowering the equilibrium nut. In any individual balance the sensitiveness increases with the time of vibration given to its beam.

3. **Resistance.**—Excessive resistance to the swing of the beam, if due to friction of moving parts, will be indicated by a failure to fulfil the two following tests, which should now be made. Note the position of the pointer on the scale at the extremes of several successive swings. The differences between successive readings on the same side will show the diminution in amplitude due to friction and to resistance of the air. This should not exceed 0.1 to 0.3 of a division in a good analytical balance. Let the balance swing until it comes to rest, and read the position of the pointer; repeat several times. The successive positions of rest should not differ by more than 0.2 of a division. Make each test with no load in the pans, and then with a moderate load (say 50 gms.) in each pan.

4. **Weighing by Swing of Balance.**—In most other tests of a balance this method of weighing is necessary, and is therefore inserted here. There are two methods of weighing a body, of which the first, and ordinarily the most convenient for moderate loads when the extreme limit of precision is not desired, is to first adjust the beam until the index swings exactly equal distances on each side of the middle division of the scale, allowing for the diminution of amplitude in successive swings; then to place the body to be weighed in the left-hand pan and weights in the right, until, with proper adjustment of the rider, the index again swings equal distances on each side of the middle point. The second method is the more precise, and is more rapid with heavy loads on slow balances of precision, if not in all cases where really extreme precision is sought. It is applicable to balances without riders, and requires no weights smaller than 1 mgr. It is as follows:

(a) Have the balances adjusted so that the index swings about equally on each side of the middle mark of the scale. Find the position of rest of the index, by observing and recording the turning points, or extreme positions, of the index at both ends of several successive swings. For this purpose it is best to have the divisions of the scale numbered from right to left, calling the middle division 10, or some convenient number, since the weights are ordinarily used in the right-hand pan, and an increase of weight thus increases the index reading. This numbering is assumed in the examples below. Swings of not over 4 or 5 divisions on each side are best, and an odd number of readings, say 5 (3 at one side and 2 at the other) or 7, should be taken to eliminate the effect of resistance.

Example.—Load = 0.

<i>r.</i>	<i>l.</i>
6.2	14.2
6.4	14.0
6.5	14.0
<u>19.1</u>	<u>28.2</u>
6.37	14.10
	<u>6.37</u>
	<u>20.47</u>

Point of rest = 10.23

This point of rest with no load will be varied by any disturbance of the level of the balance, and can only be used so long as it is known that the adjustment of the level is not disturbed. It must be redetermined frequently, and in general whenever any doubt could arise.

b. Place in the left-hand pan the body to be weighed. Put weights into the other pan, and adjust the rider (or small weights, as the case may be), until a

weight is obtained which is less than 1 mgr. too small, so that 1 mgr. additional would be too great. Suppose that in an actual case the smaller weight was 6.154 gms., the larger 6.155 gms., and the turning-points of the index with each successively were as here recorded.

Wt. = 6.154 gms.		Wt. = 6.155 gms.	
<i>r.</i>	<i>l.</i>	<i>r.</i>	<i>l.</i>
6.7	13.2	6.8	16.7
6.8	13.0	7.0	16.5
7.0		7.2	
<u>20.5</u>	<u>26.2</u>	<u>21.0</u>	<u>33.2</u>
6.83	13.10	7.00	16.60
	6.83		7.00
	<u>19.93</u>		<u>23.60</u>
Rest = 9.96 div.		Rest = 11.80 div.	

∴ deflection for 1 mgr. under this load = $11.80 - 9.96 = 1.84$ div. (This, as will be shown later, is the “sensitiveness” of the balance under this load, and may be determined from the curve or table of the sensitiveness, described in § 5, with a considerable saving of time if many weighings are to be made with the same balance.)

Now from the previous determination (*a*), it is found at the time of this weighing that with no load the point of rest = 10.23. Hence the weight which would have exactly counterpoised the substance, *i.e.*, which would have made the index again vibrate about this point of rest 10.23, would have been,

$$\begin{aligned}\text{Wt. desired} &= 6.154 \text{ gms.} + \frac{10.23 - 9.96}{1.84} \text{ mgr.} \\ &= 6.154 \quad + 0.00014 \text{ gms.} \\ &= 6.15414 \text{ or } 6.1541,\end{aligned}$$

rejecting the last place of figures if, as usual, the weighings do not pretend to a precision of more than one-tenth of a mgr. Of course any other increment of weight than 1 mgr. may be used, but it is desirable that a small increment only should be taken, since for large differences of deflection the deflection may not be strictly proportional to the load. In general, let

$$\begin{aligned}a &= \text{point of rest of index with too small weights, } w \text{ gms.} \\ b &= \text{“ “ “ “ weights greater by } m \text{ gms.} \\ c &= \text{“ “ “ “ no load.}\end{aligned}$$

Then desired weight in gms.,

$$W = w + \frac{c - a}{\frac{b - a}{m}}$$

By this method make a weighing of a piece of metal provided in the balance in this experiment. Record the mark upon the metal. All records of weighings should be complete, and should be kept in regular form.

5. **Sensitiveness.**—By the sensitiveness of a balance is meant the difference of index reading produced by the addition of 1 mgr. to one pan of the balance. If the beam were perfectly inflexible, and the three knife-edges in the same plane, the sensitiveness of the balance would be the same, whatever the load (supposed always equal in the two pans) on the balance. But as inflexibility is impossible, the sensitiveness of the balance must vary with the load, diminishing as the load increases.

To study the sensitiveness of the balance, place equal weights in the two pans. Exact counterpoising is unnecessary. Determine as in (a) the point of rest a . Then add on one side (r supposed), by means of the rider, a weight of 1 or 2 mgrs. = m , and again find the point of rest b . Then the sensitiveness *with this load* (by "load" is always meant weight in one pan only) is

$$\frac{b-a}{m}.$$

Find the sensitiveness for each 10 gms., from 0 up to 50 gms. (or up to the limit of the balance load, taking care not to injure the balance by excessive loading). Plot points with loads as abscissas and sensitiveness as ordinates, and draw a line through them. This curve or a table of the determined sensitiveness and corresponding loads may be used for interpolation in weighing, as suggested in (b), and may thus considerably facilitate work.

6. **Ratio of Arms of Balance.**—In general, of course, there must be a greater or less difference between the length of the balance arms, *i.e.*, of the distance of the outer knife-edges from the centre one. This distance is usually adjusted by the balance maker, so that the ratio of the balance arms shall be correct within the limit of precision for which the balance is intended; and, in many cases where only relative weighings are desired, an error in this ratio is of no consequence. But, where accurate weighings are to be made, a test must always be had of the ratio of the arms. Call

R = length of right arm of balance,

L = length of left arm of balance,

and suppose weighing always to be made with the weights in the right-hand pan. Then any weight w thus found to be sufficient to counterpoise the substance to be weighed must be multiplied by the ratio $R \div L$ to reduce it to the true weight, which would be found if the balance were equal-armed.

If two bodies or weights known to be precisely equal (within the limits of precision desired) were at hand, then the ratio could be found by placing one of these bodies in each pan, and then adding weights to one or the other pan until the pointer oscillated about the previously determined point of rest with no load. Suppose that a weight w_r is necessary in the right pan; then, by the laws of equilibrium of moments,

$$(W + w_r) R = WL;$$

whence,

$$\frac{R}{L} = \frac{W}{W + w_r},$$

where W is the weight placed in each pan. If the extra weight required is in the left-hand pan, then $w_r = 0$, and a corresponding term w_l must be added to the numerator.

But in general the nominally equal weights of a set are not, in fact, sufficiently nearly so for the present purpose, and the procedure must be as follows:—

Let A and B be two weights or combinations of weights nominally equal, and, in fact, very nearly so; and in amount, each about half the maximum load for which the balance is designed. Place A in the right pan, and B in the left. Add weights to the apparently lighter pan until the index vibrates equally about the observed "point of rest with no load." This small weight is best added by the rider. If various small weights are used instead of the rider, care must be taken that they are in proper adjustment, lest they introduce error. Suppose a small weight r to be required in the right pan. Then reverse A and B , and suppose a small weight l to be required in the left pan for equilibrium. If these small weights require to be in the reverse pans from those mentioned in either or both

cases, it is simply necessary to account them negative weights in the pan above named. Then, by the law of moments,

$$R(A+r) = LB \text{ and } RB = L(A+l).$$

Solving by eliminating B gives

$$\frac{R^2}{L^2} = \frac{A+l}{A+r},$$

which, by two approximations, since l and r are small referred to A , gives

$$\frac{R^2}{L^2} = 1 + \frac{l-r}{A} \text{ and } \frac{R}{L} = 1 + \frac{l-r}{2A},$$

in which the observed values of l , r , and A may be substituted.

In obtaining the small weights r and l necessary for the equilibrium, the method of using the swing of the balance, as described in (b), may be employed to advantage instead of equal oscillations about the point of rest.

Example.—Maximum load of balance 100 gms.

<i>Right.</i>	<i>Left.</i>
50 gms.	(20 + 20 + 10) + 0.0030 gms.
(20 + 20 + 10)	50 + 0.0005
$\therefore r = -0.0030$	$l = +0.0005.$
$\therefore \frac{R}{L} = 1 + \frac{0.0005 + 0.0030}{2 \times 50} = 1.000035.$	

7. Errors of Set of Weights.—Weights from the best makers can seldom be relied upon as correct to 1 mgr., and, in precise work, it is therefore necessary to determine the relative errors of the weights of the set in terms of some one weight of the set; and further, if necessary, to find the absolute error of this weight by comparison with a standard weight. The method for this study of the weights may be found in Kohlrausch's *Physical Measurements*, p. 32, and in Thresh's *Experimental Physics*, p. 25, preferably the latter. Students will omit the work of this paragraph unless personally instructed to perform it.

METEOROLOGICAL INSTRUMENTS.

This experiment includes the reading of the Fortin and Siphon barometers, self-registering thermometers, the wet and dry bulb thermometers, the anemometer, etc.; and its object is to familiarize the student with these instruments.

Barometers.—A description of the Fortin barometer may be found in Ganot's *Physics*, p. 122, and in Deschanel, p. 147. To read the barometer, turn down the screw at the bottom of the cistern until the mercury stands slightly below the ivory point projecting downwards from the top of the cistern. This insures a rising meniscus at the top of the column in the subsequent setting. Then turn the screw up until the ivory point just touches its image in the mercury surface when the barometer is stationary. If this position be passed, the point will produce a depression in the mercury surface. Many observers prefer to turn up the mercury until a very minute depression is obtained, instead of observing simply the coincidence of the point and reflection. This method has some advantages, and is in general more accurate. When this is done, jar the tube near the top of the mercury column by tapping lightly upon the side with the hand. This tends to secure uniformity in the meniscus at different readings, eliminating the effects of unequal adhesion of the mercury to the walls of the tube. The vernier is arranged to slide up and down by a rack-and-pinion movement connected with the milled head at the right-hand side of the tube. On the

opposite side of the tube from the vernier is another similar metal plate which moves with the vernier, and has its lower edge in the same plane, perpendicular to the tube, as that of the vernier. If, therefore, the eye be raised or lowered until just in line with these edges, the line of sight will be perpendicular to the tube. This is very important. The setting is made by turning the vernier up until wholly above the mercury column, then turning it down, the eye being kept as nearly as possible in the plane just described, until the edges, when in line, appear just tangent to the top of the meniscus, *i.e.*, just cutting off light at the point *a* in the figure, while allowing it to pass through at the sides. While



FIG. 8.

making this setting, the barometer should be turned toward a bright background. The vernier of the best English and American barometers, having a scale of inches, usually reads to 0.002 of an inch, and by estimation to 0.001 of an inch.

As the column of mercury at higher temperatures required to produce the same pressure is greater than at lower ones, owing to the less density of the mercury, it becomes necessary to reduce the reading to a standard temperature. The thermometer on the front of the column has its bulb inside the brass tube, and gives quite accurately the temperature of the mercury; but when the changes of temperature are rapid, it becomes essential to surround the brass tube with a heavy wrapping of woollen material, to render changes in the air temperature less quickly felt by the mercury; otherwise, the whole column will not be at the same temperature, and the indication of the thermometer will be of uncertain precision.

This precaution is necessary whenever an accuracy of 0.005 inch is sought. The Signal Service directions for reading the barometer require the reading of the "attached thermometer" before that of the vernier. The standard temperature adopted in scientific work and by the U.S. Signal Service, to which to reduce all observations is $0^{\circ}\text{C}=32^{\circ}\text{F}$. As the barometer scale is usually of brass, this reduction may be made by dividing the reading at a temperature t by $1 + \gamma t$, where γ = the coefficient of apparent expansion of mercury on brass, $= 0.000181 - 0.000019 = 0.000162$ per degree Centigrade. The correction to be subtracted from the direct reading is, however, more conveniently found from tables such as are given at p. C, 63, or p. C, 79 of the "Smithsonian Collection." (The numbering referred to is at the bottom of the page.) An explanation of



FIG. 9.

the use of each table is given at its beginning. The figure shows the principle of the siphon barometer, and an inspection of the instrument will render the method of reading obvious. The graduation is numbered each way from the centre, and the sum of the two readings thus gives the difference of level of the upper and lower meniscus.

The third form of mercurial barometer among the meteorological instruments is known as the "Kew Standard." In this the area of the tube bears a known ratio to that of the cistern, and the scale is graduated so that each inch shall be shortened by that amount; thus the rise of the mercury in the cistern is compensated without the adjustment necessary in the Fortin barometer; but the instrument, although convenient, is less accurate than the Fortin.

Read each of these instruments and the attached thermometers, and reduce each reading to 0° by the tables. Reduce the metric barometer reading to inches. (1 inch = 25.400 mm.)

Calculate by Guyot's tables, "Smithsonian Collection," p. D, 33, the height of a place at which the barometer reading is 1.111 inches lower than the reading obtained by the Fortin, calling $t = \tau$ = the observed temperature by the attached thermometer, $t' = \tau' = 55.5^{\circ}$, and $\text{Lat.} = 35^{\circ}$.

Thermometers.—In Rutherford's maximum thermometer a small piece of iron may be brought by a magnet, or by shaking, into contact with the top of the mercury column, and will then be pushed forward as the mercury expands, but will be left behind by the retreating mercury column as the bulb cools, and its lower end will thus indicate the maximum temperature obtained. In the Rutherford minimum thermometer a glass index is placed within the alcohol thread. It is drawn back by adhesion and capillary action as the top of the thread retreats, and is left behind as the column advances, thus recording by its upper end the minimum temperature attained. The index is adjusted by tapping on the thermometer when in an inclined position. The Negretti and Zambra maximum thermometer is made with a contraction in the capillary near the bulb. When the mercury in the bulb contracts, separation takes place at that point, and the column remaining in the capillary indicates the maximum temperature. The reading should be taken with the tube slightly inclined downward toward the bulb, as an inspection of the instrument will show. By tapping the end of the instrument against the hand, or by a whirling motion, the thread may be forced back into the bulb. Read each thermometer, heating by the hand or breath if necessary to observe its action.

Moisture of the Air.—The amount of vapor of water in the air may be determined by several methods, of which the direct methods of measuring the weight of some substance having a great affinity for water, before and after having been exposed to a known volume of moist air, are those upon which the others, more convenient in use, are based. The method in most common use is known as that of the "Wet and Dry Bulb Thermometer."

The moisture of the air is usually expressed either by, 1st, the weight of water contained in the form of vapor in each unit volume; 2d, the ratio between the amount actually present to that which would be present if the air were saturated at the temperature of observation. The second is the more common, and the more generally useful method, and this ratio is called the "relative humidity" or "humidity" of the air. It is expressed in percentages. Thus relative humidity = 75 per cent or 0.75 denotes that the air contains three-fourths of the amount of moisture required to saturate it at the same temperature.

The use of the Wet and Dry Bulb Thermometer is based upon the fact that water at the temperature of the air evaporates less rapidly as the humidity is greater; also that, in evaporating, the water absorbs heat from the bodies with which it is in contact. A glance at the apparatus will show its construction. By direct experiments it has been ascertained what difference of humidity corresponds to one degree difference between the two thermometers for any given temperature of the air. Upon these determinations have been based tables which are used in connection with the instrument. (See Ganot, 290-7, Deschanel, Stewart.) Observe upon the Negretti and Zambra Wet and Dry Bulb Thermometers the reading of each, taking pains not to warm either during observation. Turn to the "Psychrometrical Table," by Glaisher, Smithsonian Coll., p. B, 102, and by aid of the description of the table find the temperature of dew-point, force of vapor, weight of vapor, and relative humidity. Guyot's table, p. B, 46, is probably more accurate, but does not exhibit these quantities quite as clearly. Compare these results with those taken from the tables of the U.S. Signal Service which are placed with the apparatus. The quite large discrepancies are due to the variations in the data upon which the tables are constructed. For greater convenience several graphical and other devices have been introduced into similar instruments to render the reference to tables unnecessary. One of these is called the "Hygrodeik." Set this instrument as

directed upon the back of its plate, and read from it as there directed the four quantities above mentioned. Owing partly to errors in the thermometers, the results from this will not closely agree with those previously obtained.

Anemometer.—Instruments for measuring the velocity of the wind are known as anemometers or air meters. One of the most perfect forms consists of a wheel carrying inclined fans, against which the wind impinges and causes rotation. This wheel is connected through others to hands moving over a dial, upon which the velocity may be read off. A test of each instrument should be made, giving a correction to be applied to readings. A detent on the side of the instrument serves to throw the wheels in or out of gear. To make an observation, throw the wheels out of gear, and take the reading of the hands. Hold the instrument in front of some ventilating register or in any air current. At the beginning of a minute again throw the wheels in gear, and at the end of the minute throw them out of gear. The difference from the former reading will give the velocity in feet per minute. Calculate the velocity in miles per hour, which is the time usually employed as unit in meteorology.

Of the other forms of anemometers in use, the "cup anemometer" (used by U.S. Signal Service) is the most common. At the end of each of four arms at right angles to each other are placed hemispherical cups, as shown in horizontal section in the figure. The wind coming in the direction of the arrows will meet with less resistance from the convex surface than from the concave one, and rotation will be set up in the direction of the curved arrow.

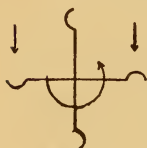


FIG. 10.

COMPOSITION OF FORCES.

Place the blocks to which the spring balances are attached at any convenient positions on the hoop. With the weight removed from the vertical cord, and with the cords wholly slack, read each balance. This reading will be the index error of the balance, and must be added to or subtracted from all subsequent readings with the balance in that position, according to whether it is negative or positive. This index error must be determined for each position of the balances.

Attach the 5-lb. or 10-lb. weight to the third cord. Turn the adjusting screws at the top of the balances until the cords meet at the pin passing through the centre of the circle. Read the spring balances, and record these and all other observations at suitable places on a sketch showing the direction of the cords. Find the exact direction of the three cords by moving the fine thread attached to the pin in the centre of the graduated circle until it is parallel to each cord in turn, taking the corresponding reading on the circle to tenths of a degree.

Considering either pair of the three forces acting, then the other must be equal in magnitude and opposite in direction to the resultant of this pair.

Make the necessary observations with three or more positions of the balances. Find for one set of these, by geometrical construction to scale, the resultant of two forces, only one of which is a balance reading.

From the triangle of forces it follows, that when three forces are in equilibrium about a point, these forces are to each other as the sines of their opposite angles in the triangle. Therefore, in any one set of the observed results, the ratio of each force to the sine of its opposite angle should be the same. Compute the value of each ratio in all the cases tried.

DIVIDING ENGINE.

Paragraphs 1, 2, 3, and 4 of experiment 21, as described in the "Physical Manipulation," p. 56, are to be performed.

1st. To measure the back lash. Focus the microscope carefully for any very fine and distinct point upon the glass plate. Any minute scratch or irregularity of surface is suitable. Bring the cross hairs up to this point from the right-hand side, taking pains that the motion of the screw is always in one direction, so that the nut is bearing up on one side against the threads of the screw. Read the position of the micrometer. Then turn the screw until the cross hairs are upon the left-hand side of the point, and bring the cross hairs then up to the point from this (left) side of it, taking pains as before that the screw is turned in one direction only, so that the nut is now bearing against the screw thread in the opposite direction from that in the first setting. Read the micrometer. The difference of these two readings will give the extreme amount of play of the nut upon the screw at this part of the screw; that is, will give the "back lash." Take a number of settings in each direction, and compute the "Average Deviation" of the setting.

2d. For measuring the pitch of the screw, use the steel scale of inches provided with the apparatus, and find how many revolutions of the micrometer, reading always to 0.001 revolution, correspond to an inch upon the scale. Let this be n revolutions. Then the pitch of the screw is $\frac{1}{n}$ of an inch. The measurement thus made is, of course, not of a high degree of precision, owing to the kind of scale used, and no temperature correction need therefore be applied.

3d. Measure the distance between two successive pairs of crosses upon the glass plate, recording the number of the crosses used.

4th. Measure the two co-ordinates of a series of points (not less than seven) on the larger smooth curve on the plate of acoustic curves and plot the curve.

CRANK MOTION.

These notes supplement the description at p. 63, "Physical Manipulation." First measure AD , as may readily be done with sufficient accuracy without removing it from its place. Then measure CD by reading A with F first at 0° and then at 180° . The zero-point 0° should be in line between C and A . To test whether it is so, and to find the error, set F exactly at 90° . Read A . Turn F until A reads again the same, when F will have come to, or nearly to 270° . Then 270° minus the actual reading of F will be twice the index error of the circle, which must be allowed for in setting F in line with C and A , and in all the subsequent settings at 10° , 20° , etc. The explanation of the last correction will be seen by inspection of the exaggerated sketch in Fig. 12. Only one length of arm CD is to be used in the experiment.

In computing "the distance of AB from the mean position," CD must of course be expressed in the same units as those in which the reading at A is taken. Also it is to be noticed that the "mean position" referred to is the mean between the reading of A when F is 0° and when F is 180° . The observed distances of A from this mean position are obtained by subtracting from the observed readings of A this mean position reading.

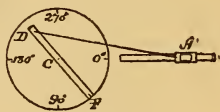


FIG. 11.

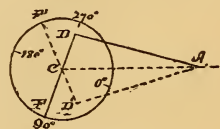


FIG. 12.

ANGLE OF FRICTION.

Read pp. 71, 72, "Physical Manipulation."

In the apparatus used in the laboratory, the distance from the hinge F to the left-hand edge of the scale rulings is one metre, and the scale is numbered in decimals of a metre, so that the readings where the wire index crosses this edge give tangents of the angle DAB directly. Now the weight of the body E may be represented by the vertical line om , Fig. 13; this force may be decomposed into the two others, on , perpendicular to AD , and mn , parallel to it. The former, on , produces pressure on AD ; the second, mn , tends only to make E slide along AD . The coefficient of friction of motion is the ratio

of mn to on , when the angle DAB is just sufficiently large to have E continue in uniform slow motion when once started. But the ratio $\frac{mn}{on}$ is the tangent of mon , and $mon = DAB$; hence the coefficient of friction is $\tan DAB$, which, as before stated, is given directly by the scale reading. In the experiment, take five readings for each of the three substances furnished, determining the coefficient of motion only, and take the mean of each series. It will be noticed that with the same substance the block will move with a different velocity, according to the angle used. Take pains, therefore, that the readings are always taken when the block moves with the slowest attainable velocity. Always start the block from the reference mark across AD , and let it slide a short distance only so that it may not change to an entirely new part of the surface. Set the block in motion by tapping it gently.



FIG. 13.

BREAKING WEIGHT.

With this apparatus the breaking weight, or the stress required to break wires of iron, copper, etc., is to be directly measured, and from this and the diameter of the wires broken, the "tenacity" is to be calculated. The "tenacity," also known as the "ultimate strength," "modulus of rupture," and "resistance in tearing," is usually expressed by the weight which is just sufficient to break a rod of unit cross-section when applied in the direction of the length of the rod. If the weight be expressed in pounds, and the area of cross-section in square inches, then the tenacity would be in pounds per square inch; that is, would be the number of pounds weight which, when applied longitudinally, would be just sufficient to tear apart a bar of 1 sq. in. cross-section. This quantity may of course be also expressed in grammes per sq. centimeter, or kilogrammes per sq. millimeter (a commonly employed unit), or, following the absolute system, in dynes per sq. centimeter.

Cut off a piece of wire of the length indicated by the marks on the front of the apparatus. Twist one end of this piece around one of the pins in the crank at the right-hand end of the frame, first raising the wedge C (Fig. 14) nearly out of the frame, so that the balance shall be strained to but one or two pounds. Pass the wire around the hook at B , so that it shall lie in the groove cut for it, and then attach the other end to the second pin in A . In both the attachments at A the wire should be twisted around the pin on the lower side, and pass between the upper projections. By this arrangement the wire is not sharply

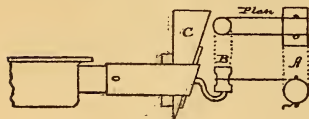


FIG. 14.

bent at any point where the greatest strain occurs, and is therefore not weakened by the method of attachment. After the wire is fastened in place, turn the crank slowly so as to wind the wire upon it, and see that the wedge slides freely down as the balance spring is stretched. When the wire breaks, the wedge will retain the balance at the reading at which it stood when the wire broke, and this reading will be twice the breaking weight, since the wire is double and each strand carries but half the total pull. Record in one column the diameter of the wire before breaking, as measured to ten thousandths of an inch by the micrometer gauge, in the next column the diameter near the point of rupture after breaking, and in the third the observed balance readings. Break three samples of each kind of wire provided. Calculate from the mean of each set of three the tenacity of that wire.

Let d = diam. after breaking,

w = weight required to break a single strand; then

$\frac{\pi d^2}{4}$ = area of cross-section of wire, and

$\frac{w}{\frac{\pi d^2}{4}} = \frac{4w}{\pi d^2}$ = tenacity = wt. req. to break a wire of unit cross-section.

Rankine gives for wires of

Iron, tenacity = 70,000 to 100,000 lbs. per sq. in.

Copper, tenacity = 60,000 lbs. per sq. in.

Brass, tenacity = 49,000 lbs. per sq. in.

Remembering that

1 lb. = 453.6 grammes,

1 sq. in. = 6.452 sq. cm.,

calculate the tenacity of the iron wire in grammes per sq. cm., in kilogrammes per sq. mm., and in dynes per sq. cm.

DEFLECTION OF BEAMS. I.

Supplementary to pages 77, 78 of "Physical Manipulation."

In this experiment make readings as follows:—

1st; page 77, line 2 from below. Find deflection for every hundred grammes from 0 to 1 kgr., carefully remove the load from the pan, and take the readings *in each case* with no load as the zero reading for that load; and in the calculations use for each deflection its corresponding zero reading. Be careful to turn the micrometer screw up before removing the load, otherwise the platinum contact surface upon the beam will become indented and injured.

2d; page 78, line 7. Perform in full to line 21. Substitute for the remainder of the paragraph the following, and omit the last paragraph on page 78.

The deflection a of the centre of the beam is

$$a = \frac{Wl^3}{4Eb\delta^3},$$

where b = breadth of beam = 6.73 mm.,

δ = depth of beam = 3.73 mm.,

l = length of beam between supports,

W = weight applied at middle of beam, and

E = modulus of elasticity of the material = the ratio between the weight applied longitudinally to a rod of unit cross-section and the elongation thereby produced.

Since all these quantities are known except E , this may be calculated. Substitute the observed value of a when $W=1$ kgr., and compute E . In computations, all dimensions must be expressed in terms of one unit. If this be the millimetre, E will be in kgr. per sq. mm. Calculate E in this unit; then, noting that 1 kgr. = 2.205 lbs., and 1 sq. mm. = 0.00155 sq. in., calculate E in lbs. per sq. in. Rankine gives in the latter unit, for steel bars, $E=29,000,000$ to $42,000,000$. The steel (Stubb's steel) used in this experiment has a lower value than this.

DEFLECTION OF BEAMS. II.

In a beam of uniform cross-section, imbedded at one end and loaded at the other, the greatest angular deflection is given by the expression $i = \frac{1}{2} \cdot \frac{Ml}{EI}$, where M = moment of load referred to supported end A of beam,
 l = length AB ,
 E = modulus of elasticity of the material of the beam, and
 I = moment of inertia of its cross-section.

This formula is derived on the hypothesis that the deflection is so small that the arc is sensibly equal to its tangent, and is, of course, only true within these limits. Using the gravitation system of measuring forces, $M=Wl$, E = the ratio between the weight applied longitudinally to a rod of unit cross-section and the elongation thereby produced, I for a rectangular cross-section = $\frac{1}{12}$ of the product of the breadth b into the cube of the depth d of the section;

$$\therefore I = \frac{bd^3}{12};$$

$$\therefore i = \frac{1}{2} \cdot \frac{Wl \times l}{E} \cdot \frac{12}{bd^3} = 6 \cdot \frac{Wl^2}{Ebd^3}. \quad (1)$$

Similarly the vertical deflection of the end B of the beam will be

$$a = \frac{1}{3} \cdot \frac{Wl^3}{EI} = 4 \cdot \frac{Wl^3}{Ebd^3}.$$



FIG. 15.

In the apparatus used, the telescope is to be placed opposite the mirror, and so adjusted that the reflection of the scale is plainly seen, and that OB from the scale to the mirror is about one metre. If the beam be then loaded at any point, as C , the mirror will be tipped forward until its new position makes with its former position an angle i , which, since the portion CB of the beam remains unbent, is evidently the same as that which the surface of the beam at C makes with its original position. The angle OBN will be twice this angle; hence $\frac{ON}{OB} = \tan 2i$, the line of sight OB being always in the same direction. Now, as i is necessarily limited to very small values, the arc is sensibly equal to the tangent; hence

$$i = \frac{\tan 2i}{2} = \frac{ON}{2OB}. \quad (2)$$

Take the scale reading when no load is applied. This gives the zero error by which subsequent readings are to be corrected. Place the weight of 1 kgr. at the point marked 50 cm., and read the point where the intersection of the cross-hairs appears on the scale. Put on 2 kgr. at the same point, and read the deflection. The angle i should be twice the former, i.e., the scale readings

should be as 1 : 2. Calculate their ratio. Repeat at the 30 cm. point. Then place 1 kgr. successively at the 10, 20, 30, 40, and 50 cm. points, and take the corresponding scale readings ON , observing first in each case the reading with no load. From (1) it appears that the angular deflection should be proportional to the square of the length of the beam, *i.e.*, of the distance AC ; hence, by (2), if a series of points be plotted with abscissas proportional to the squares of 10, 20, etc., and ordinates to values of i ; or, since i is directly proportional to ON , with ordinates representing the successive values of ON , these points should lie along a straight line. A slight variation from a straight line may arise from imperfect rigidity in the supporting pieces at A .

From the data thus obtained, the modulus E may be calculated, since in (1) all the remaining quantities are known, assuming $b = 0.83$ cm. and $\bar{d} = 0.818$ cm. Insert then in (1) the values of arc i , W , l , b , and $\bar{d}$, when $l = 50$ cm. (using same unit of length in all cases), and calculate E . The ordinary value of E is about 20,000 kgr. per sq. mm., but the steel (Stubb's steel) of the beam used here has a lower modulus.

LAW OF PENDULUM.

Give to the string which suspends the ball such a length that the ball shall hang near the floor. Measure the distance from the point of suspension to the top of the ball, expressing the result in *metres* and decimals. Add to this half the diameter of the ball, obtaining this by calipers. The sum of these would give nearly the length of equivalent, simple pendulum, but the centre of oscillation is, in fact, slightly below the centre of the ball, and the actual length desired will be found by adding to this sum $\frac{2}{5} \frac{r^2}{h+r}$, where r = radius of the ball. Set the ball swinging through an arc of 10° or 15° only, and find time required to make 50 or 100 single vibrations. Deduce from this the time of a single vibration. Repeat with several lengths of pendulum, the shortest being not much less than a quarter of a metre.

If T is the time of a single oscillation, and L the length of the pendulum, then $T = mL^n$. Find, from observations just made, the values of m and n by application of the "logarithmic method." The complete equation would be

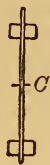
$T = \pi \sqrt{\frac{L}{g}}$; whence it appears that the observations should give $n = \frac{1}{2}$ and

$m = \frac{\pi}{\sqrt{g}}$. From the value of m found from the curve, calculate g . The metre is used as the unit of length in order to bring the axis of Y among the observed points, so that m may be accurately determined. If any other unit were used, the intercept would lie considerably beyond the observed points, as will be seen on inspection. Students who have not studied the graphical method will compute g from each length of pendulum used by the formula $t = \pi \sqrt{\frac{L}{g}}$.

METRONOME PENDULUM.

To prove the law of the simple pendulum by means of the metronome pendulum, we make the latter of such form that from the known dimensions and weights of its parts we can calculate the length l of the equivalent simple pendulum, *i.e.*, of the simple pendulum that would oscillate in the same time as the

metronome pendulum used. Then, by comparing l and t , we may find the relation between them. Suppose the pendulum to consist of movable weights, w_1 and w_2 , sliding on the rod AB . Let the distance of the centre of gravity of each from the knife-edge C be fixed for an experiment at l_1 and l_2 respectively, and the rod be considered without weight. Then the length of the equivalent simple pendulum would be



$$l = \frac{w_1 l_1^2 + w_2 l_2^2}{w_1 l_1 + w_2 l_2}. \quad (1)$$

Note that l_2 , being measured downwards, is always negative, so that the denominator, although an algebraic sum, is an arithmetical difference. To correct this result for the weight of the rod,

FIG. 16.

we must add to the numerator of the fraction the term $W \left(\frac{r_1^2 + r_2^2}{4} + \frac{a^2}{3} \right)$, where W = the weight of the rod, a its half length, and r_1 and r_2 its inner and outer radii respectively. Thus we have

$$l = \frac{w_1 l_1^2 + w_2 l_2^2 + W \left(\frac{r_1^2 + r_2^2}{4} + \frac{a^2}{3} \right)}{w_1 l_1 + w_2 l_2}. \quad (2)$$

In this expression the numerator is the sum of the moments of inertia of the three parts of the pendulum, and the denominator is the sum of the statical moments, that of the rod being zero, since C is adjusted to pass through the centre of gravity of the rod. That the distance from the centre of support to the centre of oscillation is equal to the sum of the moments of inertia, divided by the sum of the statical moments, is here assumed. The demonstration may be found in treatises on mechanics. See Rankine's "Applied Mechanics," pages 516-518, and the Physics lecture notes.

For the experiment, first place both weights near the ends of the rod, taking the pendulum off the supports while adjusting the weights. Observe the time of 100 single vibrations. In making these observations, the pendulum should swing through a very small arc, say 10° or 15° only, and the time of vibration should be measured as the pendulum crosses its position of rest, not from the end of the vibration, since the slowness of the motion there renders the result less precise. Then lay the pendulum upon a table, and with a scale of millimeters measure to tenths of a mm. the distance from the knife-edge upon which the pendulum was supported to the nearest surface of each weight. Add to each half the thickness of the weight, and the results will be l_1 and l_2 . Repeat this observation with w_1 about two-thirds of the way from the end of the rod to C , and again with the lower weight also part way up to C . Deduce for each case by means of equation (2) the corresponding value of l . Substitute these values successively in the expression $t = \pi \sqrt{\frac{l}{g}}$, which gives the law of the simple pendulum, and compare the computed times thus obtained with the observed times. They should agree within about one per cent. Errors in computation will be more likely to be detected, and time will be saved by arranging the calculations and results in a systematic form.

The following data are given, but r_2 and the thickness of the weights are to be measured with calipers.

$w_1 = 355.1$ gms.	$2a = 80.20$ cm.
$w_2 = 383.3$ "	$r_1 = 0.30$ "
$W = 300.0$ "	$g = 980.4$ "

In the result l will be found negative. This denotes simply that the equivalent simple pendulum would be one whose length would be below the point of support.

BORDA'S PENDULUM.

Read "Physical Manipulation," pp. 85-87.

Determine first the time of vibration of the pendulum as there described; then remove the pendulum carefully from its support, and lay it in the frame provided for it. Take it to some light window, and measure its length as follows: Focus one of the reading microscopes upon the knife-edge, and the other upon the rod, adjusting their positions until the intersection of the cross hairs is at the knife-edge in one, and at the point where the rod enters the upper side of the ball in the other. A metre scale lying on the portion of the frame marked *A* serves to measure the distance between the cross-wire intersections in this way. The frame is moved toward the microscopes until the scale is visible beneath one of the microscopes, and points toward the other. It is too short to reach it. The scale is further adjusted until some division, *a*, is under the cross-wires. This microscope is now removed without disturbing the scale, and set down at some point further along the scale of which the reading is *b*. Without disturbing the microscopes, the scale is now moved lengthwise until it is clearly seen under both microscopes, the readings being taken as *c* and *d* respectively. Then $(a-b) + (c-d) = h$ will be the desired distance from the knife-edge to the top of the ball. Or three microscopes may be used. The diameter of the ball is now to be carefully determined by making several measurements, by means of calipers, of the vertical diameter. Call one-half this distance *r*. All these measurements are to be made as nearly as possible to tenths of a mm. The distance from the knife-edge to the centre of oscillation of the pendulum is then

$$l = h + r + \frac{2}{5} \cdot \frac{r^2}{h + r}.$$

From this compute the value of *g*, by means of the formula for the time of vibration of a pendulum. As explained in the notes for the metronome pendulum, *I* is the moment of inertia divided by the statical moment, and the above expression may then be deduced. See Rankine's "App. Mech.," pp. 516, 517, and 518, and the Physics lecture notes.

In the apparatus as arranged, the error which would arise from the moment of inertia of the rod carrying the ball has been eliminated by adjusting the distance of the small ball above the knife-edge to such a point that the system consisting of this ball and the long rod vibrates in as nearly as possible the same time as it is intended the pendulum shall vibrate when the large ball is in place. This ball is then put on and adjusted to the proper position by trial.

As the rate of the clock pendulum is liable to variation from day to day, its rate must be determined from a ten-minute observation, finding exactly the number of minutes and seconds by the dial corresponding to just ten minutes by the watch. Let *m* be the observed number of seconds by the watch equal to *n* by the seconds pendulum. Then one beat of the supposed seconds pendulum would be $\frac{m}{n}$ seconds. After having computed, as described in the text-book, the rate of vibration of the Borda's pendulum, that rate must be multiplied by this ratio, $\frac{m}{n}$, to reduce it to true seconds. The value of the fraction $\frac{m}{n}$ must be computed to the fourth place of significant figures in order to correspond in accuracy with the remainder of the work, and more precise rating than that proposed would be preferable.

TORSION PENDULUM.

A heavy cylinder of metal is supported by a thin steel wire, which, by its elasticity, can maintain torsional vibration in itself and the cylinder for a considerable time. The cylinder carries a pointer which moves over a graduated circle, and serves to measure the amplitude of the torsional swing of the pendulum. A clamp screw moving along a vertical rod near the wire can be screwed to the rod and to the wire, thus fixing the length of the wire, that portion only being under torsion which is between the clamp screw and the point where the wire becomes rigidly attached to the cylinder.

The time of vibration of the pendulum is independent of the amplitude of the vibration. Twist the cylinder, so that when swinging the pointer shall move through an angle of about 45° , 90° , 180° , and 360° successively. Note in each case the time of 50 or 100 single vibrations. The time should be very nearly the same in all cases with the same length of wire.

The time of vibration is proportional to the square root of the length of the wire under torsion. Give the wire successively four different lengths, varying from the longest attainable to about one quarter of that length. Measure in each case the length of wire under torsion, and the time of 100 single vibrations. Since the time is proportional to the square root of the lengths, the quotient $\frac{t}{\sqrt{l}}$ should be the same in all four cases. Compute and compare this ratio. State the extreme error in percentage.

In timing vibrations, the times should be taken as the pointer passes its position of rest, not from the turning point, since the slowness of motion at the latter point renders the observation less precise.

VOLUME OF FLASK.

I. To determine the volume of the glass flask. Weigh the flask empty. Fill it with water so that the bottom of the meniscus is just opposite the reference mark on the neck. Weigh again, and subtract. The difference gives the apparent weight w of the water in air. Find by a thermometer the temperature t of the water at the time of weighing. Also, read the barometer, and call H the reading in inches. The water as weighed in the air is buoyed up by an amount equal to the weight of air displaced. This weight is proportional to the volume and to the atmospheric pressure H . The volume of w grammes of water is, nearly enough for this correction, w cubic centimeters. Now one cubic centimeter of air at the ordinary temperature of the room, and under a pressure of 30 inches, weighs 0.00128 gms. Hence the weight of air displaced by the water will be $0.00128 \times w \times \frac{H}{30}$. The brass weights are, however, buoyed up in the same way; but, since their specific gravity is about 8, their volume is only $\frac{1}{8}$ that of the water. Hence, the total correction for buoyancy will be only $\frac{7}{8} \times 0.00128 \times w \times \frac{H}{30}$ gms. This added to w will give the weight P of the water in vacuo; and hence, from the definition of the gramme, the volume of the flask will be $V = \frac{P}{D}$ cu. cm., where D is the density of water at the temperature t , as given in the tables at the end of these notes. All weighing should be performed by Borda's method of double weighing. See p. 19, "Physical Manipulation."

II. Determine similarly the volume of the small bottle, but use mercury instead of water. As before, $V = \frac{P}{D}$. Let $D = 13.596$ = density of mercury at 0°C. ; then its density D at $t^\circ \text{C.}$, will be $D = \frac{D^\circ}{1 + .000182 t}$. In this case the weight of the air displaced by the brass weight is greater than that displaced by the mercury, so that the apparent weight of the latter is too great. Calculate the proper correction for buoyancy, assuming the sp. gr. of the brass as 8 and of the mercury as 13.6. Calculate also what would be the volume of the bottle at 0°C. , assuming .000026 as the coefficient of expansion of glass per degree Centigrade.

In filling the bottle with mercury, fill it first completely, then force the stopper firmly into place, allowing the overflow to run into the dish. Be careful before weighing that no globules remain attached to any part of the bottle, especially around the edge of the stopper. In using mercury avoid bringing into contact with gold finger-rings, etc., as they become amalgamated and injured.

Density of Water.

t	D	t	D
0°	0.99987	16°	0.99899
4°	1.00000	18°	0.99861
8°	0.99988	20°	0.99821
14°	0.99929	22°	0.99778

SPECIFIC GRAVITY OF SOLIDS.

The specific gravity of a substance (solid or liquid) is its weight in vacuo divided by the weight in vacuo of an equal bulk of distilled water taken at its maximum density, *i.e.*, at 4°C. As the substance whose density is to be determined varies in density with change of temperature, its specific gravity should be referred to some standard temperature. For this, either 0°C. or 62°F. ($= 16.7^\circ \text{C.}$) are ordinarily selected, the former being in general preferable when the coefficient of expansion of the substance is well known; but when this is not the case, the latter (or frequently $60^\circ \text{F.} = 15.6^\circ \text{C.}$) is better, since this temperature correction is thus rendered much smaller, and may often be wholly neglected. The standard temperature used should always be specified in stating results. It is often sufficient, however, to omit the reduction, merely stating the temperature at which the determination was made.

In the determination of specific gravity of a solid, four weighings and two temperature measurements are necessary.

W = weight to mgs. of bottle filled with water at temperature t° .

W_1 = " " " " + solid and water at temperature t_1° .

S = " " " solid dry in air.

b = " " " bottle.

D = density of water at t° .

D_1 = " " " t_1° . } To be taken from tables at the end of these notes.

k = mean coefficient of cubical expansion of glass, assumed to be the same at all temperatures, and approximately $= 0.000026$ per 1°C.

Computation of Weight at t_1° . — As the temperature t° in general is not the same as t_1° , it becomes necessary to compute from W what this weight would have been had these been the same. Call this computed weight a_1 . Assume, for

clearness, that t_1° is $\left\{ \begin{smallmatrix} \text{higher} \\ \text{lower} \end{smallmatrix} \right\}$ than t° . Then the capacity of the bottle on being heated from t° to t_1° would be $\left\{ \begin{smallmatrix} \text{increased} \\ \text{diminished} \end{smallmatrix} \right\}$ by its thermal expansion in the proportion of $1:1+k(t_1-t)$ when t and t_1 are not widely different. The exact expression is $\frac{1+kt_1}{1+kt}$. The weight of water contained in this increased volume, if the density remained the same, would therefore be

$$(W-b)[1+k(t_1-t)]; \quad (1)$$

but, in fact, the density $\left\{ \begin{smallmatrix} \text{decreases} \\ \text{increases} \end{smallmatrix} \right\}$ from D to D_1 when the water is $\left\{ \begin{smallmatrix} \text{heated} \\ \text{cooled} \end{smallmatrix} \right\}$ from t to t_1 . Hence, the weight of the water at t_1° would have been

$$(W-b)[1+k(t_1-t)]\frac{D_1}{D}, \quad (2)$$

and of the bottle and contents, this plus the weight of the bottle, *i.e.*,

$$a_1 = (W-b)[1+k(t_1-t)]\frac{D_1}{D} + b. \quad (3)$$

Reduction of Weighings to Vacuo.—The apparent weight of a substance (in air) is less than its true weight (in vacuo) by the weight of a volume of air which is equal to its own volume less the volume of the weights used.

Corrections for this loss of weight are required to the apparent weight of the solid ($=S$), and to the apparent weight of the equal bulk of water ($=w_1$) only, since all the other apparent weights cancel, and the error is thus eliminated. This apparent weight w_1 , in grammes, is

$$w_1 = a_1 - (W_1 - S). \quad (4)$$

Let A be the weight of 1 cc. of air at the temperature and barometric pressure in the room at the time of the weighing. These must be observed when W and W_1 are taken; and in the following correction all weighings are supposed to be made under the same conditions of pressure and temperature, as otherwise the correction becomes somewhat less simple. Also, the humidity of the air is neglected, as of slight effect. Tables of the weight of 1 cc. of air at various pressures and temperatures are given at page 27.

The volume of the "equal bulk of water" weighing w_1 grammes is (nearly enough for present purposes) w_1 cc.; that of the counterpoising (brass) weights is, therefore, $\frac{w_1}{8.5}$. The volume of the solid is, of course, w_1 cc., and that of the counterpoising weights is $\frac{S}{8.5}$.

Then, calling the specific gravity of brass 8.5, the loss of weight of the water owing to the buoyancy of the air is (nearly enough for the purposes of this correction)

$$\left(w_1 - \frac{w_1}{8.5}\right)A = \left(1 - \frac{1}{8.5}\right)Aw_1; \quad (5)$$

that of the solid is

$$\left(w_1 - \frac{S}{8.5}\right)A = \left(1 - \frac{S}{w_1} \cdot \frac{1}{8.5}\right)Aw_1. \quad (6)$$

Now, adding (5) to w_1 gives w , the weight in vacuo of the bulk of water at t_1° , equal to the volume of the solid at the same temperature; and adding (6) to S gives the weight, which will be called s , of the solid in vacuo.

Specific Gravity referred to Water at t_1° .—The ratio $\frac{s}{w}$ would now give the specific gravity of the substance at the observed temperature t_1° , referred to water at the same temperature. But specific gravities are properly referred only to water at its temperature of maximum density, viz. 4° C.; hence the

Reduction to Water at 4° C.—Water weighing w grammes at t_1° C., and having the density D_1 , would have a volume $\frac{w}{D_1}$ cc. at t_1° ; and, since at 4° C. its density is unity, this volume, *i.e.* this number of cc. of water at 4° , would weigh $\frac{w}{D_1}$ grammes. Hence, the desired

Specific Gravity of Solid at t_1° , referred to Water at 4° ,
will be

$$sp. \text{ gr.} = s \div \frac{w}{D_1} = \frac{sD_1}{w}. \quad (7)$$

If now it is desired to give the sp. gr. referred to some standard temperature t_2° (any desired temperature, *e.g.* 0° C.), it is necessary to know the mean coefficient of thermal expansion of the solid from 0° to t_1° and from 0° to t_2° . Call these β_1 and β_2 , respectively; then the desired Sp. Gr. at t_2° is given by the proportion

$$\frac{Sp. \text{ Gr.}}{sp. \text{ gr.}} = \frac{1 + \beta_1 t_1}{1 + \beta_2 t_2}, \quad (8)$$

the specific gravities being inversely as the volumes of the same mass. Ordinarily β_1 and β_2 may be assumed to be the same, but it should be observed that a definite and determinable precision is necessary in β , in order that a greater error be not introduced in this last reduction than in the previous parts of the actual determination.

Weight of 1 Cubic Centimeter of Dry Air.

	730 mm.	740 mm.	750 mm.	760 mm.	770 mm.
	grms.	grms.	grms.	grms.	grms.
10° C.	0.00120	0.001215	0.00123	0.00125	0.001265
12° C.	0.00119	0.001205	0.00122	0.00124	0.001255
14° C.	0.00118	0.001195	0.001215	0.00123	0.001245
16° C.	0.001175	0.00119	0.001205	0.00122	0.001235
18° C.	0.001165	0.00118	0.001195	0.001215	0.00123
20° C.	0.001155	0.001175	0.00119	0.001205	0.00122
22° C.	0.00115	0.001165	0.00118	0.001195	0.00121
24° C.	0.00114	0.001155	0.001175	0.00119	0.001205

Density of Water.

10° C. . . .	0.99974	15° C. . . .	0.99915	20° C. . . .	0.99827
11° C. . . .	0.99965	16° C. . . .	0.99900	21° C. . . .	0.99806
12° C. . . .	0.99955	17° C. . . .	0.99884	22° C. . . .	0.99785
13° C. . . .	0.99943	18° C. . . .	0.99866	23° C. . . .	0.99762
14° C. . . .	0.99930	19° C. . . .	0.99847	24° C. . . .	0.99738

SPECIFIC GRAVITY OF LIQUIDS.

This may be found by the specific gravity flask, Fig. 31, as follows:—

Let b = weight in grms. in air of bottle empty.

W = weight in grms. in air of bottle and distilled water at t° C.

W_1 = weight in grms. in air of bottle and liquid at t_1° C.

D_t = known density of water at t° C. (See tables on p. 27.)

A = weight 1 cc. of air at temperature, pressure and humidity of that in balance room at time when W is found; A_1 = ditto for W_1 (p. 27).

Reduction to Vacuo.—The weight of the flask need not be reduced, since W , W_1 , and b are very nearly equally affected, and thus the omission of the correction would not affect the differences $W-b$ and W_1-b sensibly unless b were unusually large and A and A_1 considerably apart.

The weight of water at t° filling flask at t° has a volume of about $W-b$ cc., and therefore displaces $(W-b)A$ grms. of air, while the counterpoising weights displace $A(W-b) \div 8.5$ grms. of air, 8.5 being the sp. gr. of the brass weights. Hence the corrected weight of this water at t° is

$$w = (W-b) \left(1 - \frac{A}{8.5} \right) + (W-b) A. \quad (1)$$

And, similarly, since the volume of the liquid is the same $(W-b$ cc.) as that of the water, its corrected weight is

$$w_1 = (W_1-b) \left(1 - \frac{A_1}{8.5} \right) + (W-b) A_1. \quad (2)$$

Reduction to Water at 4° C.—The weight of a volume of water at 4° C., equal to the volume of the bottle at t_1° (t and t_1 must be known to about 0.1°), may be found from w , as at pp. 26 and 27, as

$$w [1 + \beta(t_1 - t)] \div D_t. \quad (3)$$

The *Sp. Gr.* of the liquid at t_1° C. referred to water at 4° C. would therefore be

$$Sp. Gr. = \frac{1}{w} \cdot \frac{w_1}{1 + \beta(t_1 - t)} = \frac{w_1}{w} \cdot \frac{D_t}{1 + \beta(t_1 - t)}. \quad (4)$$

This formula is the simplest which can serve where weighings are made to 1 mgr. *unless* $t_1 = t$, or *unless* A and A_1 do not differ by more than 0.00001 grms., about, in which case the terms $(W-b)A$ and $(W-b)A_1$ in (1) and (2) become constant. If a less degree of precision is desired, the expression may be correspondingly simplified. *E.g.*, if the results are desired to 0.1 or 0.2 per cent. only, then the following reductions may be made: Substituting in (1) and (2) the value $A = 0.00122$ grms., which may now be assumed as constant, gives

$$w = (W-b) \left(1 - \frac{0.00122}{8.5} \right) + (W-b) \times 0.00122 = 1.0011 (W-b), \text{ and}$$

$$w_1 = 0.9999 (W_1-b) + 0.00122 (W-b), \text{ whence}$$

$$Sp. Gr. = 0.9988 \frac{W_1-b}{W-b} \cdot \frac{D_t}{1 + \beta(t_1 - t)} + 0.0012 \text{ (approx.).} \quad (5)$$

In this expression the weighings of W_1 and W are taken to 0.005 grms., t_1 and t are to be measured to 0.3° C., and β may be assumed as = 0.000026. If a series of determinations of specific gravities is to be made, the process of computation may be still further simplified, since $W-b$ and t will be constants. In results by either of the formulæ (4) or (5) the sp. gr. determined is obviously for the observed temperature t_1 *only*. This temperature must, therefore, always be stated. Reduction to standard temperature may be made as in (8), p. 27.

MARIOTTE'S LAW (a).

(PRESSURES GREATER THAN 1 ATMOSPHERE.)

The apparatus consists of three glass tubes, *A*, *B*, *C*, of about equal dimensions, terminating at the top in glass cocks, and connected at *D* by a long rubber tube to a larger glass tube at *E*, which is open to the air, and slides upon a vertical rod carrying a scale, being clamped at any desired height by a set screw. The dried gases to be experimented upon are enclosed in *A*, *B*, *C*, respectively, and may be subjected to various pressures by raising or lowering *E*. The direct object of the experiment is to determine the rate at which the volume of the gas diminishes as the pressure increases, or, in other words, the relation between the volume *v* and the pressure *p* when the temperature remains constant. Mariotte's Law asserts that the pressure

is inversely as the volume, *i.e.*, $p = \frac{c}{v}$; $\therefore pv = c$, where *c* is a constant.

This law, however, holds true only approximately in any case, and is more deviated from as the gas is nearer its point of condensation. In the apparatus to be used the tubes contain dry air, ammonia gas = NH_3 , nitrous oxide = N_2O , the two latter being the more readily condensible. The tubes are labelled with the name of the enclosed gas.

Suppose *E* to be in any position, as represented in the figure, and the mercury to stand at any corresponding height in *A*, *B*, and *C*, *e.g.*, at *a* in *A*. Then the pressure *p* upon the enclosed gas (which, as there is equilibrium, must equal the expansive pressure of the gas upon the top of the mercury column at *a*, etc.) is measured by the mercury column equal to the difference of level between *E* and *a* plus the atmospheric pressure upon the top of the column at *E*; *i.e.*, by the difference of level of *E* and *a* plus height of barometer at the time. The tubes are numbered upon the etched scale in cm. from above downward. If, therefore, from the reading of *E* be subtracted the difference between *O* and *o*, the difference will be the vertical distance from *o* to *E*; and if to this be added the reading *a* (= *oa*), this sum will be the vertical distance *aE* desired, to which must be added the height *H* of the barometer. In order to avoid negative quantities, it is better to perform these operations in the following order: $OE + a - Oo + H = p$. As the zero points on the tubes could not all well be adjusted to the same height, the value of *Oo* for each has been determined by levelling, and is as follows: *A*, *Oo* = 85.24 cm.; *B*, *Oo* = 87.05 cm.; *C*, *Oo* = 82.30 cm. Compare these figures with those given on the apparatus. The value of *p*, as well as of *v*, has, of course, to be calculated for each gas at each position of *E*.

To find the volume *v*. If each tube were of uniform cross-section throughout, and the zero point of the scale were rightly placed, then the scale reading would give the volumes directly in arbitrary units whose value would be the capacity of one unit length of the tube. Neither of these conditions can be fulfilled, and it therefore becomes necessary that the tube should be calibrated beforehand, *i.e.*, there must be found beforehand by measurement the volumes up to different marks upon the scale. This has been done for each of the tubes, and the results are shown in the curve accompanying the apparatus. To find the volume corresponding to any reading *a*, look out in the horizontal axis the number *a*, and find the ordinate of the proper curve at that point. The sum of this ordinate and the scale reading *a* gives the volume *v* in an arbitrary unit similar to that just described.

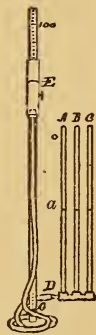


FIG. 17.

The experiment consists in placing E first as low as is consistent with having the mercury in A , B , and C come upon the graduation, and in reading all four columns; then raising E about 50 cm., reading again, and so on, taking five or six different positions of E . The barometer should be read before, during, and after the experiment, to obtain proper values of H . The meniscus at E is to be read by raising and lowering the eye in front of the tube until it appears to be on a level with the horizontal upper surface of the mercury, and taking the scale reading to tenths of a mm. The meniscus in each tube is best read by bringing the black-paper clasp down nearly to the top, but not covering the division next above it, and then bringing the eye up nearly to the plane of the lower edge of the paper, and estimating the tenths of a division (mm.) by the eye.

It is essential that the temperature should remain very nearly constant during the whole experiment; hence any handling of the tubes is to be avoided. No temperature corrections are to be applied, as for present purposes they would be unnecessary, unless a considerable change of temperature occurs during the observations.

If Mariotte's Law held true, then, as before stated, $pv = c$. To show how the gases vary from this law, calculate for each observation upon each gas the value of pv . Plot points with values of p as abscissas, and of pv as ordinates. The resulting lines will be found in each case very nearly straight, but with differing inclinations. The line would be straight and parallel to axis OX if the gas followed Mariotte's Law, and the greater the deviation the greater the inclination of the line. Find, therefore, the tangent of the inclination of the lines to OX . A negative value of this tangent indicates, of course, that the gas is more compressible than the law indicates. As the volumes of the gases are not numerically equal, the values of the tangents will not be strictly comparable. If, however, the tangent for each of the more compressible gases be multiplied by the ratio of the pv for air to pv for that gas, these products will be directly comparable.

As systematic work is of material assistance in reducing the labor of computation, and avoiding numerical and other mistakes, the student should use especial care in the arrangement of these computations.

MARIOTTE'S LAW (b).

(PRESSURE LESS THAN 1 ATMOSPHERE.)

A graduated tube il , closed at the top filled with dry air, slides with slight friction in a guide de . The enclosed air, in virtue of its expansive force, exerts a pressure upon all parts of the enclosing vessel. Hence, at the top a of the mercury column, in order to bring about the equilibrium which there exists, there must be an upward pressure exerted through the mercurial column equal in amount to the downward pressure of the gas upon the column. This upward pressure comes from the pressure of the atmosphere upon the mercury in the cistern at c , and its amount is measured by the height of the barometer at the time of the experiment minus the column ac , since the latter opposes the atmospheric pressure. The barometer column and ac must, of course, be measured in, or reduced to, the same unit. There is thus an enclosed mass of air in ai , whose volume and pressure can be measured and varied by sliding de .



FIG. 18.

Raise de until i touches the stop above the top of the tube. Read the position of the top of the meniscus at a to tenths of the smallest division

of the graduation. To do this, lower the paper clasp, which will be found on the tube, until it is nearly tangent to the top of the meniscus, thus cutting off the troublesome reflections from the meniscus, and then, looking as nearly as possible perpendicularly to the tube, estimate the fraction of the division by the eye. Call this reading a .

To find the height ac , set the rod with its lower point just in contact with the mercury at c , and read the position of the upper end b upon the scale to tenths of a division. This reading may be best made by moving the eye until the image of the rod formed by reflection from the mercury surface is just in the line of sight across the end b of the rod itself, and then reading the position of this image, calling this reading b . A white paper may be held so that its reflection will make a good background for the image of the rod. Then withdraw the rod, place it alongside the tube, and read its length bc to one-tenth of a scale division. Then, as the scale is numbered downwards from i , $ac = bc + (b - a)$.

Subtract the distance ac from the height of the barometer taken at the time (expressed in the same unit), and the difference will be the pressure p of the gas. Next, to find its corresponding volume v . This is given approximately by the scale reading of a ; but this requires a correction for irregularities of the tube, which is given by the "curve of calibration" accompanying the instrument. To find the correction, look out on the curve the abscissa corresponding to the scale reading, and find the ordinate of the curve for that abscissa. Add this ordinate to the scale reading, and the sum will be the volume v of the gas, expressed in an arbitrary unit about equal to the average volume contained between two successive cm. marks on the tube.

Determine the values of p and v for six or eight different positions of the tubes, changing the positions as much as the apparatus will allow.

By Mariotte's Law, p varies inversely as v ; $\therefore pv = c$, where c is a constant. Hence, if a curve be plotted with values of p as ordinates, and of v as abscissas, it should appear as part of a rectangular hyperbola. If a curve be plotted with values of p as ordinates and reciprocals of v as abscissas the line should be straight, and the straightness will serve to test the accuracy of the results.

Draw both curves on the same sheet.

BUNSEN PHOTOMETER.

These notes replace the description on pp. 135-138, "Physical Manipulation," but the paragraphs there given under "Apparatus" furnish some further details in regard to various parts and accessories of the photometer which are not used in this experiment.

The Bunsen photometer is a convenient device for the measurement of the ratio of two lights of the same, or nearly the same, color. A disk of white paper is covered with grease except over a central circle of about an inch diameter. This disk is placed in a line joining the two lights whose ratio is to be measured, and is perpendicular to that line. Viewed from one side, the central spot of the disk will appear brighter than the surrounding portions when the illumination of that side is greater than of the reverse, darker than the surrounding portions when the illumination is less than of the reverse. Thus, by moving the disk forward and back between the two lights, it will be found that at some point the spot and surrounding portions will appear of the same brightness, and will be indistinguishable from each other, or nearly so, the total disappearance of the light depending upon the complete equality in color of the two lights, and on the nature of the disk, as well as upon the equal illumination. When this disappear-

ance of the spot is attained, it is known that the disk must be equally illuminated on either side by its respective light. Then, since the intensity of illumination at any distance from a given source of light is inversely as the square of the distance from that source, the ratios of the two lights must be directly as the squares of their respective distances from the disk. In the photometer, as ordinarily used for gas measurements, the lights are placed at opposite ends of a bar 100 inches in length, upon which the disk slides by means of car or carrier of some convenient form. If a scale of inches be placed along this bar with the zero at one light, and if a be the observed reading of the disk, then the ratio of the light L at the further end to that C at the zero end of the bar will be

$$\frac{L}{C} = \frac{(100 - a)^2}{a^2}.$$

Instead, however, of a scale of inches, we may have one graduated to give the above ratio directly. To make this graduation, we have but to put for $\frac{L}{C}$ successively 1, 2, 3, 4, etc., and any intermediate values desired, and solve for a , marking the point thus found upon the bar. In the apparatus used for this experiment both scales will be found, but at least one result should be computed from readings on the scale of inches. In most measurements a "standard" sperm candle is placed at the zero end of the scale, and the light of this candle is taken as unity, so that the ratio becomes $\frac{L}{1}$, *i.e.*, gives the "candle power" of the light measured, the necessary corrections being applied as given below. The apparatus must, of course, be located in a completely darkened room, the walls of which should be dull black to avoid reflections. Any auxiliary lights used for reading must be small and carefully screened from the disk.

In the present experiment, the candle power of illuminating gas burning at various rates in an argand burner, is to be measured in terms of the corrected standard candle, with a view to obtaining both the nominal candle power of the gas under standard conditions, and the relative efficiency of the burner used when consuming gas at various rates.

Candle.—The standard candle is a superior quality of sperm candle, six to the pound, and so proportioned as to burn nominally 120 grains of wax to the hour. The light from such a candle, when burning at this rate in open air, is tolerably uniform and reproducible, and is assumed as the unit of light in most photometric measurements. It is further assumed, for correcting for unavoidable irregularity in the rate of consumption, that the light given out by the candle is proportional to the amount of wax burned per minute. This assumption is, however, not strictly true; and these variations in efficiency, or illuminating power of the candle, make it an unsatisfactory standard, though perhaps the best yet generally available for the purpose. Owing to this inequality in ratio, it is necessary to weigh the candle, unless some means is at hand for using the average light, or getting an average result by the use of a large number of candles. The form of balance often used for this purpose carries the candle on a short arm of the beam, and has for the other arm a longer rod notched in grains. The end of this arm carries a pointer or index, which moves over a vertical scale. This form of balance, though sensitive to perhaps the tenth of a grain, is not capable of weighing thus closely on account of the small distance between the notches; and the use of the scale at the index end, intended to also indicate grains, is at best unsatisfactory. The precision of the weighings are greatest when made by noting the exact time interval of burning a whole number of grains, as described further on.

Gas.—The State law of Massachusetts requires, at the present time, that when burning at the rate of 5 cu. ft. per hour under favorable conditions, as in an argand burner with proper air supply, illuminating gas should give out a light equivalent to that of at least 15 candles. The light given out by gas under such conditions, and called its “candle power,” is nearly, but not exactly, proportional to the rate of consumption when this rate does not vary widely from 5 cu. ft. per hour. This proportion, however, does not hold when the rate is much smaller or greater than the normal for the given burner, owing to the variation in the efficiency of the burner. The rate of consumption of gas while the pressure remains constant will be found quite uniform. It is easily measured with precision by the “wet meter” provided with this experiment. The water in the meter should be kept up to the reference mark on the glass gauge. On the dial the long hand makes one revolution for every tenth of a cu. ft. of gas passing through the meter. The outer circle over which this hand travels is divided into 100 parts, and thus reads to 0.001 cu. ft. The small hands count up these revolutions, so that one division on the circle marked “one foot” indicates 0.1 cu. ft., one revolution being one cubic foot; and so on for the other circles. The gas jet in front of this meter should be turned very low.

Time in the experiment may be taken from the clock which rings warning at 7 seconds before each minute, and a gong stroke just on the minute, for convenience in the observation of the meter.

Disk.—It may be found at first somewhat troublesome to set this with precision. Repeated trials with it should be made before beginning the experiment in order to acquire facility in its use. It will be found that the disk appears slightly different according to the position from which it is viewed, and should be looked at from a definite direction, *e.g.*, at an angle of about 45° with its surface. Also, it will be found that at first different results are obtained in setting by pushing the disk away and by drawing it towards the observer; and different results on opposite sides of the disk. After some practice these differences largely disappear; but precision is increased by setting in both directions and from both sides, and taking the mean of the four readings thus obtained. During a measurement, disk observation should be multiplied as much as possible to aid in the elimination of errors of observation.

In making a measurement there are thus to be observed the time-rate of consumption of the gas and of the candle, and the position of the disk. And as the rates are both liable to change, it is desirable to make the observations on them both and on the ratio to extend over the same time interval. This is readily accomplished as follows: Light the candle and the gas, and allow the candle to come to its normal condition of burning. Then place the balance rider in a notch near the zero end of its scale, and place small shot in the scale pan beneath the candle until the index of the balance beam goes up nearly to the top of its scale. Then wait until the index (which must be kept in vibration), settling as the candle burns, swings equal distances on each side of the middle (zero) division. Note by the clock the exact time in minutes and seconds. Record this and the reading of the rider on the balance arm. Then turn to the meter and disk readings, allowing the candle to burn without attention during the ensuing five minutes. At each successive stroke of the gong, for the three or four minutes next succeeding the first balancing of the candle, read the gas meter, and in the interval set and read as frequently as possible the disk. As the end of the five minutes approaches return to the balance, move the rider up several notches (10 or more, probably), enough to again allow the index to vibrate about a point above the middle point of the scale, note the new position of the rider, wait until, as at first, the index swings equally about the middle

scale division, and at that instant again note and record the time. The difference in times thus obtained gives the interval during which the candle has burned the number of grains found by subtracting the two corresponding rider readings. From these data the rate of consumption of the candle in grains per hour can be computed. Call this c . From the meter readings may be computed the rate of burning of the gas. Call this g . Take the mean of the computed ratio obtained from the disk settings, and call this r . Then the value which would have been found for this ratio, which would then be the candle power of the *given flame*, had the candle been burning at the normal rate, would have been (under the assumption above stated)

$$L = r \cdot \frac{c}{120}.$$

Now if the gas consumption had been at the rate of 5 cu. ft. per hour, assuming the burner to have been a proper one, this L would be the desired "candle power of the gas." If the rate is other than 5 ft., but not widely different, the c. p. of the gas will be approximately given by the expression

$$L = r \cdot \frac{c}{120} \cdot \frac{5}{g},$$

and the values of L thus found corresponding to various rates g will afford a measure of the relative efficiency of this burner, and will serve equally for comparison with the same quantity deduced from other burners. As this is somewhat arbitrary, however, in the assumption of the 5 ft. rate as standard, a better way is to use as the measure of the efficiency of the burner the light given out per cu. ft. of gas burned, *i.e.*, $E = r \cdot \frac{c}{120} \cdot \frac{1}{g}$.

Measure r , c , and g for the given burner for at least four rates of burning of the gas, ranging from the lowest to the highest flame obtainable without smoking. Compute from each the value of E . Plot points, with values of E as ordinate, and of g as abscissas. If the efficiency were uniform, the line through them would be parallel to OX . Its curvature and inclination show the relative efficiency at different rates. Find the ordinate of this curve when $g=5$, and compute from this the value of L corresponding to these measurements. Students unfamiliar with the graphical method may compute L by the last expression above given for it.

ABSORPTION SPECTRA.

The light entering AB in the direction of the arrow passes through the lower half of the vertical slit S through the lens D to the prisms H, I, J, K, L , of which H, J , and L are of crown glass, and K and I of flint glass. By this combination the rays of various colors undergo dispersion, but emerge from L parallel to their original direction. Thus, on looking in at E , the spectrum of the light coming in at S

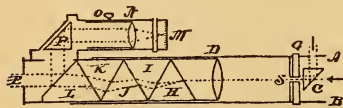


FIG. 19.

is seen. A small total reflection prism C covers the upper half of the slit S , and reflects upon it light coming in as shown by the arrow between G and B . Thus at E is seen another spectrum due to this light, and situated above the first. In the smaller side tube at M is placed a scale photographed on glass. The light coming from this scale passes through the lens N to the reflecting prism P , whence it is reflected to the surface of the prism L , and thence through E appearing below the two spectra, and forming a scale of reference. The slit

S is adjustable in width by turning the revolving rim G . Starting with the slit very narrow, first focus the lens D by drawing the inner tube back and forth until the Fraunhofer dark lines are clearly visible, pointing the spectroscopé toward a window. Direct sunlight is not necessary, and is usually too bright. Then slide O back and forth until M is clearly focused. The instrument is then ready for work.

Locate on the scale as many of the Fraunhofer lines as can be readily seen, recording their positions and their letters as found from the map accompanying the apparatus. A brief description of these lines may be found at p. 53, "Physical Manipulation," vol. i., and in the Physics Lecture Notes. Notice that these lines are most clearly seen when the slit is very narrow, and that when sunlight comes in through the front and side apertures at the same time, the various lines occupy exactly the same positions in the two spectra.

The colors of some bodies are produced by the absorption of certain portions of the light falling upon them, and this absorption may be conveniently studied with this instrument. If a mirror or piece of white paper be placed so as to throw light in the direction of the arrow at G through the prism C , the upper spectrum will be formed, and will be constantly in the field for reference. If a piece of colored glass be held in front of AB , between the window and the slit, it will absorb part of the colors of the sunlight, transmitting the remainder. The transmitted light will be dispersed by the prism, and its spectrum will appear below the reference spectrum as seen through E . In this way the transmitted light may be analyzed. Examine all the specimens of glass and the colors of the transmitted light in each case. The absorption spectra of liquids may be similarly studied.

Record the appearance of the spectra of light passed through the solutions of didymium sulphate, chlorophyll, rosin, etc., contained in the small bottles. It is sometimes more convenient to note the dark bands instead of the light transmitted. A wedge-shaped glass bottle containing a solution of potassium permanganate shows well the effect of increasing the thickness of the absorbing layer.

Finally, examine the spectra of the gelatine films between the square glass plates. These films are colored by various aniline dyes, the names of which are given on the card accompanying the plate. The best way of recording the results in this experiment is by drawing sketches of the spectra, one above another, with a scale beneath, and with the dark bands located as observed upon the scale of the instrument, placing the name of the substance opposite its spectrum.

INDEX OF REFRACTION.

The index of refraction of a glass prism may be found by the use of the optical circle described in "Physical Manipulation," page 141. In all measurements both verniers must be read, and the correction applied for eccentricity, as in the experiment on Eccentricity of Circles in these notes.

Also, if the prism be made as in the sketch, with a cavity between two of its faces, the ends of cavity being closed by suitable pieces of plate glass, the index of refraction of liquids may be obtained, observations being made as in the following directions for the glass prism.

The light from a sodium flame in front of the collimator AB passes through the slit, and emerges as a parallel beam from the lens at B , the slit having been previously adjusted to be in the principal focus of this

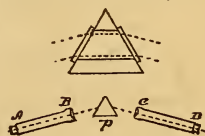


FIG. 20.

lens, as described, page 143, "Physical Manipulation." This parallel beam meets the prism, which is placed in the position shown, and emerges, after refraction, in the direction of the dotted line. This emergent beam is parallel if the faces of the prism are plane, as they should be, and is brought to the eye by the observing telescope, and thus may be seen, either with or without the latter, as



FIG. 21.

an image of the slit of the color of the sodium flame. By turning the prism alternately in one and in the opposite direction about its vertical axis, and observing the refracted image of the slit, either directly or through the telescope, it will be found that the emergent ray will move from various positions, $c'd'$ up to a limiting one cd , never approaching nearer than this to abe . The ray then undergoes, as is seen from the figure, the minimum deviation, and the angle of incidence of the ray ab is then exactly equal to the angle of emergence of the ray cd . For this position denoting by α the refracting angle of the prism, and by D the angle of deviation, *i.e.*, the angle between the line abe and cd , it may be shown that the index of refraction n is

$$n = \frac{\sin \frac{1}{2} (D + \alpha)}{\sin \frac{\alpha}{2}}.$$

To measure D , set the prism at the minimum deviation; adjust CD so that the cross-hairs bisect the image, and read both verniers. Then reverse the prism so that the emergent ray shall have the direction $c'd'$, Fig. 22. Bisect the image



FIG. 22.

as before, and read both verniers. The corrected difference will be twice the angle D . Instead of the second observation, the prism is sometimes removed after the first, and the observing telescope brought into the line nb , the direct image bisected, and the corrected vernier reading taken as a zero-reading to be subtracted from the first reading, giving D . The former method is, however, somewhat more accurate.

The value of n thus obtained is the index of refraction of the substance for the sodium line, *i.e.*, light of that wave-length. If the index for light of any other color (or wave-length) is desired, it is best obtained by finding the deviation of some suitable dark line (Fraunhofer line) in the solar spectrum, or in some cases by the use of some other than a sodium salt in the flame.

Give in the note-book a demonstration of the above formula for n .

LAW OF REFRACTION.

A slip of glass with parallel faces is mounted upon a revolving table carrying a divided circle read at a fixed index. A millimetre scale d and eye-

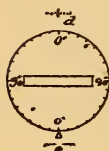


FIG. 23.

hole e are so placed that a line drawn from e through the zero of the scale would pass through the centre of revolution of the table. On looking through e , half the scale d is seen above and half through the plate. As the table is revolved in a right-handed direction, the lower half of d will appear to move to the left, so that the lower part of the divisions at the right of the middle may thus be successively brought to coincide with the upper half of the middle line. When this occurs for any division, the ray of light, coming from this to the eye, is shifted parallel to itself from its original position by an amount equal to the distance D between that division

and the middle one. Calling T the thickness of the glass (in this case 8 mm.), i the angle of incidence, and r the angle of refraction, then

$$\tan r = \tan i - \frac{D}{T \cos i}.$$

For, in Fig. 24,

$$AB = \frac{T}{\cos r} = \frac{D}{\sin(i-r)},$$

whence $\frac{D}{T} = \frac{\sin(i-r)}{\cos r}$; but $\sin(i-r) = \sin i \cos r - \cos i \sin r$,

whence $\frac{D}{T} = \sin i - \cos i \tan r$; or, $\tan r = \frac{\sin i - \frac{D}{T}}{\cos i} = \tan i - \frac{D}{T \cos i}$.

Take readings of the circle when the lower half of the first division on each side of the zero is brought to coincide with the upper half of the zero mark, the object of using both marks being to eliminate the zero error of the circle. Then turn the circle and plate through 180° , and take similar readings to eliminate eccentricity. Take the mean of this set of four readings as the value of i corresponding to this value of D . Make similar observations for each of the first five millimetre marks on each side of the middle line. Compute from the above formula the value of r corresponding to each of the five mean values of i thus found. Construct a curve, using as abscissas the sines of these values of r ; and as ordinates, the sines of the corresponding values of i . This should be a straight line, and will make with the axis of abscissas an angle whose tangent $\frac{\sin i}{\sin r} = n$ the index of refraction of the glass, thus proving the law that $\frac{\sin i}{\sin r} = a$ constant. Determine n from the curve drawn.

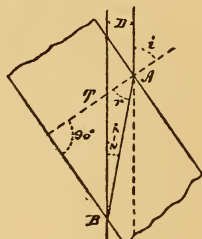


Fig. 24.

THERMOMETRY.

Each student taking this subject will,—

1st. Read carefully all the notes on Thermometry.

2d. Take a suitable thermometer, record its number in the note-book, and attach to the case a tag bearing his name, so that the same thermometer may be used in all subsequent work so far as desirable.

3d. Find its calibration errors, using a thread about 3 cm. in length.

4th. Arrange for the thermometer a suitable bath for determining the stem exposure correction, to be used in subsequent work; *e.g.*, "Expansion of Liquids," etc.

5th. Find the reading in ice, and then in steam, as described in the notes, and compute the value of α .

6th. In some cases (ask the instructor), find the deviation from the air thermometer.

The form of mercurial thermometer capable of the greatest accuracy in use is that having upon its stem an engraved scale of equidistant lines, *e.g.*, millimetres. If such an arbitrary scale is used, the corrections and reductions are in general exactly the same as for the scale of degrees, except in the stem exposure correction, where the slight difference is as shown in the notes.

The observed reading R of any mercurial thermometer is subject to several sources of error, for which the proper corrections must be applied before the

temperature of the bulb at the time of observation will be correctly known. The chief of these — to be applied in the order named — are stated in brief on this page, and in full in the following notes. The formulæ given on this page are for thermometers having scales of degrees. The formulæ for arbitrary scales will be found at the proper places in the following pages.

1st. **Calibration.** — For each reading of the thermometer, take from the curve of calibration errors (or corrections) the error (or correction) corresponding to that reading. Subtract this error from (add the correction to) the reading:

$$R' = R - e \text{ or } R + c.$$

2d. **Temperature of Exposed Stem.** — Let s° = temp. of stem, found by auxiliary thermometer; n = number of degrees length of mercury thread in stem at s° ; and 0.000156 = coefficient of apparent expansion of mercury in glass. Then

$$R'' = R' + 0.000156 (R' - s^\circ) n.$$

3d. **Reading in Ice, and Value of One Unit of Scale Division.**

R_o' = observed reading in ice, corrected for calibration.

R_s' = observed reading in steam, corrected for calibration.

T_s = computed temperature of steam. See p. 42.

The value of one unit of scale in degrees centigrade is

$$a = \frac{T_s' - 0^\circ}{R_s' - R_o'}, \quad \therefore R''' = a (R'' - R_o').$$

4th. **For Difference between the mercurial and air thermometer, and, when the limiting precision is desired, for difference between the air thermometer and the absolute scale of temperature.** This total correction will be a function of R''' for each particular thermometer, and may be represented by $\phi(R''')$.

$$T = R''' + \phi(R''').$$

1st. Calibration.

No thermometer tube of perfectly uniform bore can be obtained; hence it is essential that the scale etched or engraved upon the glass stem (a scale engraved or etched upon the tube is the only one suitable for exact physical or chemical work) should be so spaced as to mark off equal volumes of the capillary, or that it should consist of equal linear spaces, so that the points corresponding to equal volumes may be determined by calibration. Thermometers with approximately accurate scales of the first kind constitute the main supply in the market, for makers are aware that their instruments are ordinarily judged by their *apparent* and not by their *corrected* indications. Very few such thermometers are to be relied upon as possessing a maximum calibration error of less than one-half of the smallest division of the scale, and will frequently be in error from this cause by more than one whole division. Moreover, such thermometers can not be calibrated by any ordinary method to arrive at a precise determination of errors, for the divisions upon the scale consist of a series of groups of equal linear spaces, the spacing in different groups, however, being unequal. This results from the method of graduating, the stops of the dividing engine or other tool being changed only at intervals, so as to make the average error not to exceed some specified amount. In the cheaper grades of thermometers this work is very roughly done. Any thermometer with a scale of equal linear parts, whether of approximate degrees or of arbitrary lengths, may be readily calibrated by the following method so that no error exceeding 0.1 of a division shall exist uncorrected at any place. As calibration is ordinarily to be performed upon completed thermometers, such only will be now considered.

The principle of all methods of calibration consists in noting the exact length of a thread of mercury of suitable length placed successively in different parts of the capillary, and from this determining the points separating equal volumes.

To separate a thread of the desired length, the method described in the "Physical Manipulation," p. 76, may be used, or the tube may be placed in a horizontal position and tapped somewhat sharply upon the upper end with a piece of soft wood. This will cause the thread to break into pieces, from which may be made up the proper length. In order that the calibration may be carried to the lower end of the scale, it is usually necessary to remove some of the mercury from the bulb into the small bulb at the top of the instrument. As this operation is most readily understood by witnessing it, no description will be here given, but each person will be shown individually.

Method of Calibration.

The calibration correction is usually to be found for every 2 cm. to 3 cm. of the tube, the latter being sufficiently close for most work. Separate a thread of mercury about 3 cm. long, the actual length within a few millimetres being of no consequence. If the mercury should project from the bulb into the stem more than this amount, it will be necessary first to separate some of the mercury from the column, and transfer it to the auxiliary bulb at the top, so that it may not interfere with subsequent operations.

Set the thread with its lower end at or near the beginning of the graduation. It is often sufficient to start at the zero-point of graduation, however, when the scale is not to be used below that point. It is frequently best to carry the calibration thread at once to the top of the capillary, and to make the observations as the thread is moved from the top to the bottom of the tube, instead of the reverse, thus avoiding the possible inconvenience of having the mercury from the bulb accidentally enter the capillary and unite with the thread. The description supposes the readings to be begun at the lower end of the tube. Call the reading of the lower end of the thread l_1 ; that of the upper end u_1 . Move the thread less than 1 division and read again, finding thus l_2 and u_2 . Move the thread half its length or less and read l_3 and u_3 . Move less than 1 div. and read l_4 and u_4 . So continue throughout the whole length of the graduation, increasing the number of settings, and repeating the whole series in reverse order if higher precision is desired. Avoid, as far as convenient, taking readings with an end of the thread apparently just at a line of the scale, as the width of a line, even in the best scales, is a source of considerable error.

It is essential that the temperature of the capillary should be as uniform as possible throughout these measurements, and the tube must therefore be handled as little as possible during the work, and then only by its ends. In very careful work further precautions are necessary.

Now $u_1 - l_1$, $u_2 - l_2$, etc., will give a series of lengths of the calibrating thread in all parts of the tube. Before reuniting the mercury thread with the main portion, plot points with abscissas l_1 , l_2 , etc., and ordinates $u_1 - l_1$, $u_2 - l_2$, etc., the corresponding lengths of thread, and draw a smooth curve through the points thus obtained. This line will give a general idea of the form of the capillary bore, and should any part of it show considerable irregularities, the corresponding portions of the tube should be at once reexplored with the thread.

Select arbitrarily as the starting point A for the computation of errors any point near the lower end of the tube, but not so near the bulb as to be in the expanded end of the bore. Find by interpolation on the curve the ordinate u' corresponding to the abscissa A ; then, with the abscissa $A + u'$, find the corre-

sponding ordinate u'' ; with the abscissa $A + u' + u''$ the ordinate u''' , thus continuing to the upper limit of the graduation. These points, $A, A + u', A + u' + u''$, etc., upon the graduation are separated by equal volumes of the capillary, and if the tube were of perfectly uniform bore, the intervals $u_1 - l_1, u_2 - l_2$, etc., would be all numerically equal. Suppose, for convenience of explanation, that $A = 0$, and that $u_1 = 10.3$, then u_2 , if the tube were uniform in bore, should be $2 \times 10.3 = 20.6$. But suppose by the reading it is found to be 20.8, then the error at that point due to irregularity of the bore, would be $20.8 - 20.6 = +0.2$, and so on. But by this process of computation the numerical values of the errors may in some cases become inconveniently large, and this difficulty may be avoided by using as the calibration unit instead of $u_1 - l_1$ the average length of thread over a considerable length of tube. For this purpose select any one of the upper points of equal volume obtained as above (*viz.*, $A + u', A + u' + u''$, etc.), and call this point B . Then $A + u' + u'' + \dots + u^{nth} = B$. There are thus n spaces of volume equal to that of the calibrating thread in this interval $B - A$, and the average length of the thread in this part of the tube is $\frac{B - A}{n}$. Hence, the numerical error at A and at B will be zero, in the same way as at A and $A + u'$ in the previous illustration. Thus, the true reading at the point

A	is	A ,
$A + u'$	"	$A + \frac{1}{n}(B - A)$,
$A + u' + u''$	"	$A + \frac{2}{n}(B - A)$,
.....	"	,
B	"	B .

These readings, if subtracted from the interpolated readings $A, A + u', A + u' + u''$, etc., give the errors at these latter points, and these are to be computed. Thus, the error at

A	is	0,
$A + u'$	"	$A + u' - \left\{ A + \frac{1}{n}(B - A) \right\} = u' - \frac{1}{n}(B - A)$,
$A + u' + u''$	"	$u' + u'' - \frac{2}{n}(B - A)$,
.....	"	,
B	"	0.

In selecting the points A and B , it is often convenient to take for the former the zero-point of the graduation, and for the latter a point near the top of the graduation, so that n shall be as large as convenient. The result will be of equal precision in any case, but the correction will ordinarily be numerically greater when n is small than when it is large. The error or corrections ($=$ errors) are, for purposes of interpolation, most conveniently represented graphically by a smooth curve drawn through points plotted with abscissas proportional to the direct readings, $A, A + u', A + u' + u''$, etc., and ordinates proportional to the corresponding corrections.

Should it be desirable to increase the accuracy by a second calibration, with a thread of different length, it is only necessary to take one of approximately an integral part of $(B - A)$, and when the final curve of errors is drawn, make the error at B equal to zero, distributing the difference between the two curves at B among the errors at intermediate points proportionally to their respective scale readings. In other words, shift the axis of the second curve of error so that it shall make the error at B zero.

2d. Temperature of Exposed Stem.

In temperature measurements it is seldom possible to have the stem immersed in the bath of which the temperature is to be found. Thus, a portion of the mercury and stem are at a temperature different from that of the bulb. The reading of the thermometer will, therefore, require correction by an amount proportional to the apparent expansion in glass of this exposed portion of the mercurial column when heated (or cooled, as the case may be) from its actual temperature to that of the bulb.

I. For Thermometers with Scale in Degrees. — Let

R' = observed reading R corrected for calibration. (Approx. degrees.)

s° = observed temperature of exposed part of stem (In approx. degrees) by an auxiliary thermometer.

n = number of degrees of stem at temperature s° .

T = true temperature of the bulb (Desired).

0.000156 = coefficient of apparent expansion of mercury in glass.

Then the reading corrected for stem-exposure will be

$$R'' = R' + 0.000156 (T - s) n;$$

or, since T and R' are nearly equal and T is unknown, this is found approx. as

$$R'' = R' + 0.000156 (R' - s) n.$$

If the last term of the second member of this equation becomes large, it may be necessary to treat this value of R'' as a first approximation only, and, substituting it in place of R' in the parenthesis, to compute a second value of R'' . It frequently occurs that $R' - s$ is as large as 60° when $R' = 100^\circ \text{ C.}$, and $n = 100$. In such a case $0.000156 (60) 100 = 0.94$, nearly one degree. At higher temperatures the correction often becomes ten times this amount. In the determination of n the calibration correction may usually be neglected.

To obtain s the best way is to surround the stem of the thermometer with a thin glass tube about an inch in diameter, closed at the bottom with a thin section of a rubber stopper. Read the temperature of the liquid in this tube by an auxiliary thermometer placed successively at different heights in the tube. A vertically moving stirrer will generally serve to keep the temperature of the liquid sufficiently uniform, but when this does not suffice, more than one auxiliary thermometer must be employed. For the liquid, water may be used when T does not exceed 100° C. ; above that glycerine or some non-volatile transparent mineral oil. Rubber will withstand the action of a hot oil bath for some hours at 250° , but is not serviceable at such high temperatures.



A rough approximate calculation, made beforehand for any case, will FIG. 25. show whether this correction need be applied. It should be used with the high temperature thermometer in all subsequent calorimetric work.

II. For Thermometers with Arbitrary Scale. — See page 43.

3d. Reading in Ice, and Value of One Unit of Scale Division.

I. When the thermometer is capable of reading in ice and also in steam, or, as ordinarily expressed, can have the "Freezing and Boiling Points" determined, the value of the unit of scale division may be determined by reading in ice and in steam at known pressure in suitable succession as described below. This method applies equally to a scale of degrees or of arbitrary division of equal length, remembering that the calibration corrections must be applied to every reading. The reading in ice should usually precede that in steam.

To determine the "freezing point," or 0° of the centigrade-scale = 32° of the Fahrenheit scale, place the thermometer in a considerable mass of finely pounded ice, with the zero-point (a centigrade thermometer always supposed) just visible above the ice. Then pile up fine ice around the stem for a distance of an inch or more above the zero, so that the stem may be thoroughly cooled. Allow the instrument to stand for about ten minutes, then remove the ice in front of the zero-point, and read as accurately as possible the position of the mercury, taking care always to look perpendicularly to the tube. Replace the ice, and, in a few moments, repeat the observation, continuing such readings till they appear constant. Let this reading be called R_0 . Next determine the reading of the

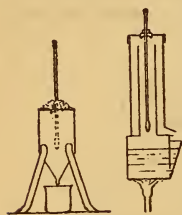


FIG. 26.

thermometer in steam, reading at the time the barometer and attached thermometer. The apparatus to be used for the readings in ice and steam are those shown in Fig. 26, and described in most text-books on Physics. ("Physical Manipulation," ii., 73 and 74.) Almost any vessel about 4 inches in diam. and 6 inches or more deep will answer for the ice, though it is better to have the water drain away.

A convenient and safe arrangement for holding the thermometer is shown in Fig. 26. The stem fits tightly into a thin section of rubber stopper, but runs loosely through a stopper, which is fitted permanently into the top of the heater. The thermometer bulb should be about two inches above the surface of the water, and within the double jacket. The water should be boiling, but not violently. The top of the mercury column must be below the stopper, as shown, so that the whole of the mercury column is in the steam, except when readings are to be made, for which purpose the thermometer may be raised until the mercury column is just visible above the stopper.

Let R_o' = observed reading in ice corrected for calibration.

R_s' = " " " steam " " "

T_s = computed temp. of steam as found from the reduced barometer reading either by tables (Smithsonian Coll. of Meteorological Tables; Landolt's Tables; Wüllner's Lehrbuch der Exp. Phys., iii., etc.) or by the approximate formula $T_s = 100 + \frac{3}{80}(H - 760)$ where H = corrected barometric height in mm.

The degree centigrade by the mercurial thermometer may be defined as that temperature interval which would produce in the mercury of the given thermometer 0.01 of the apparent expansion observed when this thermometer is heated from 0° C. to 100° C. (as elsewhere defined), the whole instrument being always at the temperature of its bulb. Then the value of one unit of the division of the scale, whether this be of approximate degrees or wholly arbitrary, will be in true degrees by the mercurial thermometer

$$a = \frac{T_s - 0^\circ}{R_s' - R_o'}.$$

If, then, R'' be any observed reading of the thermometer corrected for calibration and stem exposure, the correction for its reading in ice and for the value of the unit of the scale will make this

$$R''' = a(R'' - R_o').$$

If the thermometer is capable of reading in ice but *not* in steam, then a suitable known temperature T_h (that of a water-bath, in which the thermometer studied and a standard thermometer are immersed together) must be used in place of the steam. Then, if R_h' is the corrected thermometer reading at T_h ,

$$a = \frac{T_h - 0^\circ}{R_h' - R_o'}.$$

If the thermometer reads neither in ice nor in steam, then two suitable temperatures, T_h and T_c , must be used (determined as was T_h in the previous case). Then

$$a = \frac{T_h - T_c}{R_h' - R_c'}.$$

For thermometers of arbitrary scale, the stem-exposure correction takes the form

$$R'' = R' + 0.000156 n [(R' - R_c') a - s^\circ]$$

when the thermometer is capable of reading in ice. Note that R'' , R' , n , and R_c' are all in *divisions*, but that s° is in degrees, as this would ordinarily be taken by means of a cheap thermometer reading in degrees.

But if the thermometer does not read in ice, then the stem-exposure correction takes the form

$$R'' = R' + 0.000156 n [a (R' - R_c') + T_c - s^\circ],$$

or

$$R'' = R' + 0.000156 n [aR' - aR_c' + T_c - s^\circ]$$

when the temperature $-aR_c' + T_c$ is a constant for a given thermometer (affected only by permanent changes in the volume of the bulb). The letters have the same meaning as elsewhere. The derivation of this expression will be seen on considering that the direct expression for the correction is $0.000156 n (T^\circ - s^\circ)$, where T° is the temperature of the bulb (which is sought). The approximate value of T is $a (R' - R_c')$, when the thermometer reads in ice; while in the case in hand, where R_c' cannot be observed, $T = a (R' - R_c') + T_c$, approximately.

It will be seen that the quantity $-aR_c' + T_c$, if divided by a , would give the reading in divisions which the thermometer would have in ice if the scale were extended downwards.

The position of the zero-point is variable. It rises as the thermometer grows older, and by continued use of the instrument at high temperatures. It is temporarily lowered by sudden cooling; the amount of the lowering being greater the more rapid the cooling and the higher the temperature from which it is cooled. These two opposite effects have caused much confusion, owing to the failure to notice their independence. Continued heating at any temperature, followed by *slow* cooling, will cause a rise of zero. The amount of rise will be greater as the duration of the heating is greater. Thus, if the zero-point be determined first, the boiling point then determined, and a repetition of the zero-point determination then made, this last will be found greater than before if the thermometer be cooled exceedingly slowly; but less than before if cooled rapidly.

This might be tested as follows (the test will usually be omitted as the time required for it is excessive): 1st. Find zero-point. 2d. Immerse the thermometer bulb in a large mass of cold water (several litres), and heat this gradually to boiling. Turn out lamp and allow apparatus with thermometer in place to cool for some hours, until nearly at the temperature of the air. Remove the thermometer and redetermine the zero-point. If the cooling has been sufficiently slow, the zero will be found higher than before. 3d. Now heat the bulb of the thermometer again in boiling water, and withdraw it while hot, allowing it to cool rapidly in the air. Immerse it in ice as soon as cool (this will require less than five minutes), and find zero-point again. It will be found lower than before. Repeat this experiment, if the thermometer will admit of it, at some higher temperature, using an oil-bath.

The tendency of the bulb of the thermometer is to contract indefinitely, both by the action of the cohesion of its molecules, and by the external atmospheric pressure, the latter being the less active agent. This contraction takes place

much more readily at high than at low temperatures, and is more evident when the glass is imperfectly annealed, as after sudden cooling from a high temperature. The slowness with which the glass regains the original dimensions after sudden cooling is remarkable. The method of annealing glass is governed by these facts.

The zero-point of a thermometer is rendered rather more stable by long continued heating at high temperatures. The distance between the freezing and boiling points is probably not entirely constant, but becomes less, *i.e.*, the coefficient of expansion of the glass increases slightly with age and use. A new thermometer should, before use, be continuously heated for a long time at as high a temperature as possible, and then very slowly cooled. The freezing and boiling points may then be determined with the same result, whatever the order, *provided* the cooling is not rapid. If this heating has been once carried out, or if the thermometer has been much used, the interval from the freezing to the boiling point, or the value of 1 division of the scale, will need to be redetermined only at long intervals of time; the change of zero-point, *i.e.*, of R_0' being observed by occasional readings in ice. It is usually better to determine the freezing before the boiling point for reasons apparent from the above statement.

4th. Deviation from the Air Thermometer.

For sufficient reasons the air thermometer is accepted as the basis of temperature measurement, either directly or with the very slight reduction necessary to reduce it to accordance with the "absolute scale" of temperature. It is essential in most accurate work that the deviations of the mercurial from the air thermometer should be known; but this is only possible by a careful comparison of each mercurial thermometer individually with the air thermometer or with a standard mercurial already submitted to this study, for the deviation depends upon the rate of variation in the coefficient of expansion of the glass at various temperatures. The deviation may amount to as much as 0.5°C . at 40°C ., 2° to 3°C . at 250° , and 5° to 6° at 300°C .

The comparison with the air thermometer may be made to aid materially in eliminating some other errors of the mercurial thermometers, and must be regarded as essential in all accurate research involving any considerable range of temperatures.

SPECIFIC HEAT.

Find the specific heat of a piece of copper, or other substance, as follows:—

Weigh the substance when dry. Suspend it from the hook in the cover of the heater, inserting the thermometer T (using a high range thermometer, as this temperature might burst the calorimeter thermometer) through the hole in the cover so that its bulb comes within the substance, if possible. Students who

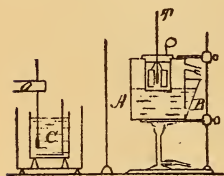


FIG. 27.

have had "Thermometry" will apply the stem-exposure correction for T , and will use the thermometer which they have calibrated, if it is suitable. Cause the water in AB (about one-third full) to boil gently. Thus the inner tube of the heater becomes surrounded by steam, which can escape through a side orifice. The temperature of this inner space will rise, at first rapidly, later more and more slowly. The substance will have nearly the temperature of its surroundings, and, when the rise

has become very slow, the substance and thermometer may be assumed to have the same temperature. This is, of course, precisely true only after the tempera-

ture has remained constant for some time, more or less long as the substance is a poorer or better conductor.

While the heating is going on weigh the calorimeter and stirrer C . Fill it two-thirds to three-fourths full of water which has been brought to within about 1° of the temperature of the room by the addition of hot or cold water, or by standing sufficiently long in the room, and weigh. Distilled water is preferable but not necessary. Set the calorimeter in place within the outer vessel, and put the thermometer and screen in position.

The quantities which must be known to determine the specific heat are, the temperature t_s of the substance before its insertion into C , the temperature t of the water in the calorimeter at the moment when the substance is to be inserted, the temperature t_m of the "mixture" when the substance has acquired the temperature of the water surrounding it in the calorimeter. As the various operations require some time, and the water in the calorimeter is in general at a temperature different from that of its surroundings, there is a certain amount of heat lost or gained by radiation, and this must be properly determined and corrected for. Also the instrumental errors of the thermometers used must be properly corrected. Note the number of each of the thermometers used, and inquire what the corrections are. These either will be supplied, or must be determined.

The order of experiment is this:—

1st. When the substance is sufficiently heated, and before its removal, insert the calorimeter thermometer (*dry*) between the outer vessel and the calorimeter, and against the inner surface of the former, and read carefully the temperature to be taken as the temperature of the surroundings t_a ; then insert the thermometer in place in the calorimeter; stir the water thoroughly (stirring must precede every thermometric reading); read the thermometer every half-minute for five minutes, recording the time in minutes and seconds direct, and the observed temperature, reading the thermometer to tenths of the smallest divisions.

2d. Read immediately the temperature of the hot substance, and withdraw the thermometer T . Transfer the substance as quickly and carefully as possible to the calorimeter, noting the minute and second at which it is immersed.

3d. Stir the water in C briskly and continuously, reading every half-minute the calorimeter thermometer, and recording, as before, the time and corresponding temperature. Continue the reading for five or ten minutes.

The observations are now complete. From them we may obtain,—

W = weight of substance.

w = weight of water in calorimeter.

w_1 = weight of calorimeter.

w_2 = weight of stirrer.

s_1 = specific heat of substance of which the calorimeter is made.

s_2 = ditto for stirrer.

x = specific heat to be determined.

t_s = corrected temperature of substance before immersion.

t = corrected temperature of water in calorimeter before insertion of substance.

t_m = corrected temperature of water in calorimeter at the time when the substance had acquired the temperature of the water.

t_a = corrected temperature of surroundings as found by the calorimeter thermometer, when dry, and placed against the inner surface of the outer vessel, before the insertion of the substance.

If calorimeter and stirrer are of same material they may be weighed together, and, of course, $s_1 = s_2$. If they are of brass or copper, $s_1 = s_2 = 0.095$.

To determine t and t_m , making correction for radiation, the method given in

the next paragraph is sufficiently precise. First correct all readings for calibration, and all readings which require it for stem-exposure. The reduction to actual degrees by the correction for the value of one division, etc., may be left until after the values of t and t_m are obtained from the method to be described, and these values then corrected, thus saving a considerable number of reductions. The process will be exactly the same, of course, with the uncorrected temperature readings (*i.e.*, in approximate degrees or in arbitrary units) as for actual temperatures which are assumed in the description.

Cooling Correction.—The temperature of the water at the start must be not more than about 1° C. from that of the surroundings, and in the method given in this paragraph it is assumed that this difference is so small as to introduce no sensible error into the correction if not considered. The complete correction making allowance for this initial difference between the air and calorimeter is given in the succeeding paragraphs. In general, water which has been standing in the room for a few hours will be sufficiently near to this condition, being, however, slightly cooler than the air. On a sufficiently large scale, not less than 0.01 inch to 0.1 of smallest scale-division of the thermometer, plot points with times as abscissas, and observed corrected temperatures of the calorimeter as ordinates. A smooth line drawn through these points will have the general form of the line $imn''o''p''r''$, Fig. 28. Find carefully the maximum point o'' of this curve. The part $p''r''$ of the line will be sensibly straight. Draw through o'' a line $o''j$ parallel to $p''r''$. Then the length of the intercept mj on the ordinate through m , the time of insertion of the hot substance, will be the rise of temperature which would have occurred in the calorimeter had no loss by cooling occurred. The smoothness of the line $mo''p''r''$ will depend largely on the thoroughness in stirring the water of the calorimeter, and the care and precision in the temperature readings. The line t_a is a horizontal line whose ordinate is equal to the temperature of the surroundings. The ordinate of the point m will give the desired temperature t , and thus the ordinate of j will give the desired temperature t_m .

Demonstration.—Suppose, for simplicity, as in the preceding paragraph, that t_a is so nearly equal to t that the calorimeter is neither gaining nor losing heat. This requires that there should be little or no evaporation from the surface of the water, otherwise the condition of equilibrium is, that the temperature of the water is lowered by evaporation to such a point that the heat received by it from the surroundings is just equal to that lost by evaporation, so that the general statement of this condition should be: suppose that the line im is parallel to OX , so that on the whole there is neither gain nor loss of heat. Then, during the time τ of reaching the maximum temperature t_o (at o''), the calorimeter will have been always at a temperature higher than that of the air, and there will have been, therefore, a continual loss of heat. This loss will have been at an average rate somewhat, but not much, less than the rate shown by the curve at $p''r''$, since the time of heating from m to n'' is in general much less than that from n'' to o'' . If this latter rate, as shown by the tangent of the angle which $p''r''$ makes with OX , be called C , then the loss during the time τ occupied in the heating from m to o'' will be somewhat less than $C\tau$. Now it must be noticed that when the maximum o'' is reached by the water of the calorimeter, the hot substance will still be at a somewhat higher temperature than t_o , since it is still giving out heat to the water at the rate C , for the rate of change of temperature of the whole system at o'' is zero. The amount of heat

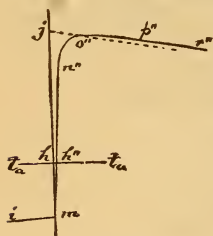


FIG. 28.

yet to be given out by the substance at this temperature is, however, small, and it may be shown by a demonstration based on Newton's Law of Cooling that this amount is almost precisely equal to the amount by which $C\tau$ is greater than the true loss by cooling as the temperature of the calorimeter rises from m to o'' . Hence, if no loss by cooling had occurred, the temperature of the calorimeter, when the temperature of the substance and water had finally become identical, would have been

$$t_m = t_o + C\tau.$$

The graphical method described in the preceding paragraph gives t_m by the ordinate of the point j . The connection between this demonstration and that graphical solution will be seen by drawing through o'' a line parallel to OX , and noting that the intercept on mj between this new line and the point j will be equal to $C\tau$, so that the ordinate at j is equal to the ordinate at o'' (i.e., t_o) plus $C\tau$.

This, however, is only for the case when the initial rate of cooling as indicated by im is zero, and the results of this method are more satisfactory the more closely that condition is fulfilled. The complete correction when C is not zero will now be given. Suppose the line t_a parallel to OX to correspond to the temperature at which C would be zero, and suppose that there is no evaporation, so that t_a is the observed temperature of the surroundings. Then, if t_a does not differ much (say more than one-third of mj) from t , the following demonstration and method will be quite satisfactory. Suppose t_a to be between t and t_o in the present case, then im will incline upwards to m , and the calorimeter will receive heat up to the time when its temperature reaches t_a . Let C' be the tangent of im with OX , as found from the plot or computed from the observation; then, until t_a is reached, the calorimeter gains heat from its surroundings at an average rate (since $h''m$ is sensibly straight, and the gain or loss of heat is proportional to the difference of temperature) of very nearly $\frac{C'}{2}$, and thus gains a total amount $\frac{C'}{2} \cdot hh''$. Moreover, the correction $C\tau$ as previously deduced on the supposition that $t_o = t_a$ makes allowance for a cooling during this time when there is now a heating, and a complete demonstration would show that this allowance was at very nearly this average rate $\frac{C'}{2}$, and in total amount nearly $\frac{C'}{2} \cdot hh''$. Hence, the total correction deduced for the former case, viz., $C\tau$, is too great for the present case by $2 \times \frac{C'}{2} \cdot hh''$, and the desired temperature would now be

$$t_m = t_o + C\tau - C' \cdot hh''.$$

It should here be noted that the signs of the rates C and C' are taken as + when they correspond to a cooling, and - to a heating from without.

In the graphical solution of the complete correction, Fig. 29, this value of t_m would be obtained by subtracting from j the quantity hw , which is the intercept on the ordinate mj , between the line t_a , parallel to OX , and the line $h''wi'$, drawn through h'' parallel to mi . Or t_m might be obtained by dropping from h'' the ordinate until it passed at x through the line im prolonged, and then drawing the line xu perpendicularly to mh ; when $uj = t_m - t$, or $t_m = t + uj$. If t_a is below m , the term $C' \cdot hh''$ becomes + instead of -, and in the graphical solution the line im will incline downwards towards m . It will be seen that so long as $t_a - t$ is small in proportion to $t_o - t$ the term $C' \cdot hh''$ remains very small, and may be dropped entirely, thus leading to the first approximate method given when $C' \cdot hh''$ does not exceed about ten per cent of $C\tau$, as the value of the latter cannot usually be determined within that degree of precision.

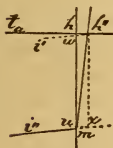


FIG. 29.

This method of correction is derived from one first given by Professor Rowland.

Specific Heat of Calorimeter.—The specific heat or water-equivalent of the calorimeter may be determined by the following method with a fair degree of precision. Of course the better way is to know the specific heat of the various materials of the calorimeter and the weights of those materials, or to determine the specific heat of a mass made up of those materials in the proper proportion.

Insert in the calorimeter enough water at about 5° above the atmospheric temperature to somewhat less than half fill it. Stir continuously, and note time and temperature each half minute for about five minutes. In a beaker have a weighed amount of water at about the temperature of the air, sufficient to somewhat less than half fill the calorimeter. Note the temperature of this water by another calorimetric thermometer from minute to minute, before taking that of the warm water as just described. At a noted time pour the cold water carefully into the warm, stirring and reading temperature of the mixture at half-minute intervals.

Plot the temperature observations as in the specific heat measurements, and find the temperature of both warm and cold water at time of mixture, and maximum temperature after mixture. The latter may be found by extending backward the line of fall of temperature after mixture to the time of mixture, since the rise of temperature was by so sudden a jump, in this case, that the loss by cooling was substantially all at, or very near the maximum temperature of the mixture. Assuming the specific heat of water to be unity, or taking the value of the specific heat from the results of the work of Regnault or Rowland, the equation can be readily deduced, and the specific heat of the calorimeter computed.

LATENT HEAT OF VAPORIZATION.

When a liquid at t° is converted into a vapor at the same temperature t° , and under a constant pressure p , a definite quantity of heat per unit weight of liquid is taken up by the liquid. This heat becomes partly potential energy, by the change of relative position of the particles of the substance in the changed state of aggregation, and is partly expended in work external to the mass of the liquid, *i.e.*, in the work of pushing back the atmospheric pressure p through the space corresponding to the increase of volume of the water expanding into steam. Let v_1 be the volume of water at t° , and v_2 that of the resulting steam at t° ; then the work done against p will be $p(v_2 - v_1)$. If the

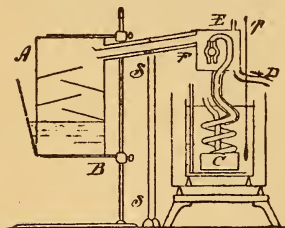


FIG. 30.

atmospheric pressure be 760 mm., so that water boils at 100° C., the total number of thermal units (kgrs. of water raised 1° C.) required to evaporate 1 kgr. of water will be 536.5; of this amount 40.2 units will be expended in work against atmospheric pressure, and hence $536.5 - 40.2 = 496.3$ will remain as energy of some sort, chiefly potential energy, in the vapor, or 92.5 and 7.5 per cent respectively. The quantity 536.5 is called latent heat of vaporization, and may be measured as in this experiment.

The steam generated in the heater AB passes into the condenser C which is open to the air. This condenser is surrounded by a known weight of water in

the calorimeter, and the rise of temperature of this is noted by the thermometer T . The increase of weight of the condenser gives the weight of steam condensed.

L = Latent heat of steam. (Sought.)

c = weight of condenser dry.

w = weight of water in the calorimeter.

w' = weight of calorimeter.

w'' = weight of stirrer.

s_c, s', s'' = specific heats of condenser, calorimeter, and stirrer, respectively.

If of copper or brass, as in this case, sp. ht. = 0.095.

W = weight of steam condensed.

t_c = corrected temp. of water just before passing the steam into C .

t_h = corrected temp. of water just after steam is cut off.

t_s = computed temp. of steam.

H = height of barometer in mm. reduced to 0°C .

The calorific capacity or water-equivalent of the calorimeter and contents will be

$$w + w's' + w''s'' + cs_c;$$

or, if w', w'' , and c are all of copper or brass,

$$w + 0.095 (w' + w'' + c).$$

The rise of temperature of this mass of material is $t_h - t_c$. The product of these two quantities will be the gain of heat, in thermal units, by the calorimeter and contents; and this must be equal to the latent heat of the weight of steam condensed, plus the heat given out by the condensed water in cooling from the temperature of the steam t_s to the temperature t_h of the water, *i.e.*, equal to

$$WL + W(t_s - t_h).$$

The temperature t_s may be determined from suitable tables (see Smithsonian Collection of Tables, Landolt's "Physicalisch-Chemische Tabellen," or Willner's "Lehrbuch der Experimental-Physik" vol. iii.), or may be computed by the approximate formula

$$t_s = 100^\circ + \frac{3}{80} (H - 760).$$

Hence, $WL + W(t_s - t_h) = (w + w's' + w''s'' + cs_c)(t_h - t_c),$

or $WL + W(t_s - t_h) = (w + 0.095 [w' + w'' + c])(t_h - t_c),$

from which L may be computed, all the other quantities being known.

To avoid, as far as possible, the introduction of steam containing condensed water, *i.e.*, "moist" or "primed" steam, the boiler is arranged, as shown in the figure, with sloping plates extending partly across its section, intended to arrest particles of water spattered from the boiling surface. The tube leading to the box EF is steam jacketed, as shown, so that the steam to be used passes only through the inner tube. The box EF serves as a steam chest, and steam is taken from it for the condenser through a cock operated by a rod through the side of the box. The entrance to the cock is downwards, so that any water condensed within it while the cock is closed may return into EF and not pass into the condenser. The connecting tube between this and the condenser is within the steam, except a short projection to which the rubber connector is attached. The outside of the whole steam space is wrapped with felting, as far as possible, to reduce loss of heat and consequent condensation and possible moistening of the steam. An exit tube is provided at E , which, ordinarily, will not be needed, and should be closed. A tube at D serves to drain away the water condensed in EF , and to

furnish an escape for the steam which does not find its way into the condenser during any part of the experiment. From D the water and steam should be led away by a rubber tube terminating in the air within some suitable receptacle. Too much resistance in the outlet will cause the water in AB to boil out through the side opening at A ; but the resistance at D must not be too small, as, in such case, little steam would flow into the condenser.

For the experiment, disconnect the condenser, and remove the calorimetric apparatus to such a position that the steam escaping from the tubes will not affect it. Start the Bunsen burner beneath the boiler AB , opening the cock in E , and allow the steam to issue through the rubber tube which is to connect with the condenser, and through the orifice D , which is to remain *open throughout the experiment*. Thorough heating of the apparatus before the beginning of the measurements is thus assured. Be careful that the water is not wholly evaporated from the boiler.

In the meantime, weigh the calorimeter and stirrer when dry. Fill about three-fourths with water, weigh again, and subtract to get the weight w of water used. If convenient, a suitable known weight of water may be readily introduced by means of a flask holding, when filled to a reference mark on the neck, the desired amount of water—a rapid method, precise to 0.5 per cent., or less, if carefully employed. Empty the condenser of water, as far as convenient, dry its outside surface, and weigh it to milligrammes. Its weight, when thoroughly dry, is marked upon it, or may be obtained from the instructor. The excess of the observed weight above that when dry is due to the water remaining within, which cannot be quickly removed. Add this excess to the weight of the water in the calorimeter, for it plays the same part as if it were a portion of that mass.

When the calorimeter is properly filled with water, close the cock in EF , continuing the boiling and allowing the steam to escape through D . Set up the screens SS . Slip on to the upper end of the condenser coil a piece of rubber tubing of such length as to extend beneath the surface of the water in the calorimeter when the condenser is in place. This will largely prevent the loss of heat in the calorimeter by evaporation of water where the hot part of the coil passes through the surface. Leave, however, about half an inch of the upper end of the metal coil free. Insert this into the rubber connector on the end of the tube from the cock in EF , having the condenser in place within the calorimeter. Put the thermometer carefully in position. Stir the water in the calorimeter continuously, and take half-minute readings of the thermometer for five or ten minutes, recording times and readings as in the experiment on "Specific Heat." At a noted time open the cock. Record temperature each half minute or minute, stirring continuously until the temperature of the water has risen about 4° or 5° . Then, at a noted time, close the cock, and continue half-minute temperature readings for ten minutes. Observe the temperature of the air by the calorimeter thermometer (dry) placed with its bulb inside the outer shield of the calorimeter; and read the barometer and attached thermometer. The temperature observations are to be treated exactly as in the experiment on "Specific Heat" to obtain the corrected rise of temperature ($t_h - t_c$) of the calorimeter. Finally, detach the condenser, dry its exterior, and weigh. The gain in weight will be the weight of the steam condensed = W . Compute L from these data.

EXPANSION OF LIQUIDS.

The discussion of Coefficients of Expansion and of the relation between the true and mean coefficient is given in the lecture notes on that subject. These should be reviewed in this connection. The mean coefficient of expansion of a liquid between 0° and t° is the average fractional increase in volume per degree from 0° to t° , the fraction being expressed in terms of the volume at 0° .

Let

v_0 = the volume of the liquid at 0° , and

v_t = " " " " " t° , some measured temperature; then

the fractional increment of volume in this temperature interval of t° will be given by $\frac{v_t - v_0}{v_0}$, and the mean coefficient of expansion of the liquid between 0° and t° will be given by

$$\beta = \frac{v_t - v_0}{v_0 t}.$$

The true coefficient at t° is the rate of expansion at that temperature. For further definition and the relation to the mean coefficient, refer to the lecture notes.

Of the various methods for obtaining the coefficient of expansion of liquids, the following is usually the most convenient when sufficient quantities of the liquids are obtainable. It has the advantages of being direct, and capable of a considerable degree of precision.

A light flask of the shape shown in the sketch is to be used. The stopper, consisting of a closed glass tube, ground into place, serves to avoid loss by evaporation during cooling and weighing. A reference mark is placed upon the narrow part a of the neck of the flask. It is evident that if the flask be weighed when containing successively enough liquid to fill it to the reference mark at various measured temperatures, coefficients of expansion corresponding to those temperatures may be determined. Let

w = weight of flask empty,

w_0 = " " " containing enough liquid to fill it at 0° ,

w_t = " " " " " " " " " t° ,

β_t = mean coefficient of expansion of liquid between 0° and t° ,

k_t = " " " cubical expan. of glass between 0° and t° .



FIG. 31.

For most liquids β varies widely with the temperature. The coefficient for glass, however, varies but little with the temperature, and may be assumed as constant and equal to 0.000026, unless the coefficient for the kind of glass used is known. To deduce the value of β from these data, call s the specific gravity of the liquid at 0° . This need not be known, as it will be eliminated from the final equation. The volume of the liquid filling the flask at 0° , and hence, of course, the volume of the flask itself at 0° , is $\frac{w_0 - w}{s}$. The liquid filling the flask at t° , and weighing $w_t - w$, would, when cooled to 0° , have a volume $\frac{w_t - w}{s}$. This volume of liquid, when heated to t° , would increase to $\frac{w_t - w}{s} (1 + \beta_t t)$, and would just fill the flask which at t° would have a volume $\frac{w_0 - w}{s} (1 + k_t t)$. Hence,

$$\frac{w_t - w}{s} (1 + \beta_t t) = \frac{w_0 - w}{s} (1 + k_t t), \text{ and}$$

$$\frac{1 + \beta_t t}{1 + kt} = \frac{w_o - w}{w_t - w},$$

whence β_t may be computed. Where k is not known from direct measurement (*e.g.*, by measuring the apparent expansion of mercury in the flask), the computation will be sufficiently precise if the approximation $\frac{1 + \beta_t t}{1 + kt} = 1 + (\beta_t - k)t$ is used, thus reducing the expression to

$$1 + (\beta_t - k)t = \frac{w_o - w}{w_t - w};$$

whence

$$\beta_t = \frac{w_o - w_t}{(w_t - w)t} + k.$$

By making a sufficient number of weighings corresponding to w_t and of measurements of t , three or more values of β_t may be computed. From these, by the method of least squares or otherwise, may be deduced the constants β_o , A , B , etc., in the equation

$$\beta_t = \beta_o + At + Bt^2 + \text{etc.},$$

which will then give the means of readily computing at any time the value of β_t , the mean coefficient of expansion from 0° to t° , for any desired temperature t . Then, from the known algebraic relation between the mean and true coefficients, the equation for the latter can be deduced, since

$$\gamma_t = \beta_o + 2At + 3Bt^2 + \text{etc.}$$

The constants for both of these equations are to be determined for some liquid; and, for the method of computation by the method of least squares, the student is referred to the notes on that subject.

To obtain the weights w_t , w_o , etc. (to mgr.) and the corresponding temperatures with sufficient precision, very careful manipulation is necessary. First weigh the flask dry. Then boil the liquid for some minutes, or heat it to a temperature above that at which it is to be used, and stir thoroughly. This serves to remove the air in solution, which would otherwise interfere with the precision of the result. If this method of removing the air is not admissible, it may be done by placing the liquid under the air-pump and exhausting. Most liquids take up air with considerable readiness, and it is therefore necessary to prevent, as far as possible, any change in the amount of air absorbed during the whole of the measurements. Fill the flask with the liquid at the temperature of the room, leaving the liquid well above the reference mark. When the neck at a is quite small, as ordinarily it should be, it is sometimes troublesome to fill the flask. In such a case, draw out a glass tube to a fine long point, insert this point downward through a into the flask, pour the liquid into the upper part of the neck, and exhaust the air from within the flask through the glass tube by the mouth, or by an aspirator. This process is much quicker than the ordinary one of heating and cooling alternately, and is preferable to putting the whole apparatus under an air-pump. Next surround the flask by finely-pounded ice, as in testing thermometers, and allow the liquid to cool to 0° , maintaining the level of the liquid above a for some minutes until thoroughly cooled. Then, by means of a fine tube, filter paper, or otherwise, draw the liquid down to the level of a while in the ice. Observe, for some minutes, to see that no further contraction takes place; and, when that point is reached, dry, as thoroughly as possible, the inside of the neck of the flask. Stopper the flask. Remove it from the ice, and allow it to stand in the air, or put it into a water bath at a temperature slightly above that of the air, until it is warm enough to weigh. Then dry thoroughly, and weigh to milligrammes, obtaining thus w_o . To obtain w_t at any temperature

below 100°C. , a double water bath may be used. This consists of two cylindrical vessels, a smaller within a larger one. At temperature above that of the air, the outer bath may be maintained at any nearly constant temperature desired by means of a flame, and the inner temperature will thus be still more constant. The liquid in the flask should be drawn down to a after the inner temperature has remained as nearly constant as possible for five minutes or more, and the temperature carefully determined, using all the necessary corrections. For temperatures of very nearly 100° , the flask may be placed within steam, as in testing thermometers, and the actual temperature may be computed from the corrected barometer reading at the time. Observations of w , should be made for at least three temperatures besides that of 0° if the equation is to contain the term Bt^2 , and for at least four places if the next term Ct^3 is to be retained, and so on. An additional observation will serve to determine k if necessary, though this is seldom done; but, of course, the precision of the constants is increased by taking more numerous measurements. It will be seen, upon consideration, that since the liquid, when weighed, is always at very nearly the same temperature, the corrections for displaced air affect the different weights in proportion to those weights, except in so far as the density of the air varies; and, as β is computed from a ratio of these weights, and not from absolute weights, all these corrections are eliminated, except in cases where the variation of density of the air would affect the result beyond the limit of precision otherwise attainable, which rarely occurs.

EXPANSION OF GASES.

The apparatus used for this experiment is a convenient form of air thermometer. A glass bulb B is connected by capillary glass tubing ("barometer tubing") with a larger tube at C , which is connected through a stop-cock D and a rubber tube to an open tube A of the same diameter as C . Both A and C are adjustable in height upon the vertical support by clamp screws. The expansion of the air in B is measured indirectly by the change of its pressure when its volume is maintained constant, and the bulb changes temperature. To measure the coefficient of expansion of the air, the bulb is first immersed in ice and afterward in steam.

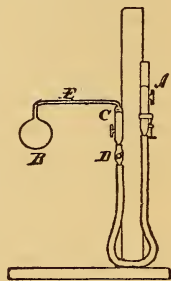


FIG. 32.

The three-way stop-cock at D may be understood from Fig. 33. Nos. 1 and 2 show the vertical section and the front of the cock in the same closed position. There are four possible closed positions, viz., as shown in 1, and 90° , 180° , and 270° from this position. The flattened or marked side of the head of the stop-cock is on the same side as the lateral boring of the cock, and the handle of the cock is parallel with the through boring. Whenever the handle or rod is at an angle of 45° to the horizontal all borings are closed. Position 3 shows the only proper position to connect the bulb B with the mercury column AD . The cock should never be left in this position, but should always be closed, except when the mercury is in process of adjustment.

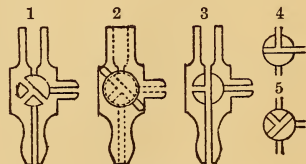


FIG. 33.

The air in the bulb must be thoroughly dried. This is facilitated by having a point, or fine tube, left at the lower side of B when this is made. The process

of filling would then be to connect this fine tube, by a rubber tube, with the drying apparatus containing fused calcic chloride and sulphuric acid, and to connect the side opening of *D*, in position 4, with an aspirator or air-pump. The bulb *B* should be kept at about 100° , and dry air drawn slowly through it for some minutes. The bulb should then be allowed to cool with *D* closed but the fine tube open; and, when at about the air temperature, the fine tube should be sealed in a flame, and the mercury, formerly standing just at *D*, should be forced up into *C*, and the cock closed. If *B* is not provided with such a tube, it must be filled by repeatedly exhausting the air from it while in steam, and allowing dry air to enter. This may readily be done by connecting the side opening of *D* with a glass *T*, one arm of which goes to an aspirator or pump, and the other to the drying tubes. A pinch cock on the rubber connectors of each branch serves to shut off the drying tubes while the exhaustion is going on, and to shut off the pump while the dry air is allowed to enter *B*. Care must be taken to introduce no dust with the air. After repeated exhaustions, the final filling of *B* should take place, with the bulb at about the air temperature, and the mercury should then be forced up into *C*. Whenever the apparatus is left with *D* closed, the column at *A* should be high enough to make the pressure outward at *D*, and thus to avoid any introduction of air by leakage.

1st. Place in the apparatus provided for this purpose a layer of finely-broken ice. Then place this upon the supporting stand at a suitable height beneath *B*, and lower *C* into place, having *D* closed throughout this and all other similar manipulations. Do not attempt to adjust the ice-box up to *B*, but always lower *C* into place. Place in position the screen between the box and the mercury columns. Carefully cover the bulb, to the depth of an inch at least, wholly with fine ice, covering the top of the ice with a flannel, or other cloth, to prevent air currents, etc. In about five minutes, the air in *B* will have attained the temperature of the ice. Then lower *A* until the mercury surface is about on a level with the top of *C*; open *D* slightly, and observe whether the mercury in *C* rises or falls. Move *A* either up or down, as may be necessary, using the vertical adjusting screw for the final setting, until the top of the mercury meniscus is exactly opposite the reference mark, near the top of *C*; the cock being open, of course, during this adjustment, so that the air pressure within the bulb is exactly counterbalanced by the atmospheric pressure and difference of level of the mercury columns *A* and *C* without.

When this adjustment is finished, close the cock *D*, and take the readings of the columns at *A* and at *C* by whatever means is provided with the apparatus used. For readings closer than 0.1 mm. a cathetometer must be used. Read the temperature of the thermometer hanging upon the apparatus, which will give approximately the temperature of this mercury column and of the small volume of air above *C*; call this *t*. Reduce the observed difference of level of *A* — *C* to the freezing point $\left(h_1 = \frac{A - C}{1 + 0.000181t} \right)$, and call the reduced difference *h*₁. This assumes the scale used to be correct at about the temperature of the air. If the cathetometer be used, the instrumental errors and coefficient of expansion to be used must be requested from the instructor. Read the height of the mercurial barometer and the temperature of the attached thermometer. Reduce this height to the freezing point by the tables, and call the reduced reading *H*₁. The temperature of the air in the tube above *C* is *t*₁^o, while that in *B* is 0^o. The correction for this difference is as follows. Let *v* (obtained as described further on) represent the volume of the air above *C* at *t*₁^o, then its volume at 0^o would be $\frac{v}{1 + \alpha t_1}$, where α is the coefficient of expansion of air. This air is, of course, at a pressure of *H*₁ + *h*₁, as is also the air in the bulb.

Care must be taken that the mercury never falls below the lower part of the tube C , and that it is not forced over into the bulb. If the former occurs, the experiment must be repeated, owing to the loss of air; if the latter happens, ask for instructions. The cock D must be closed at all times, except when the final adjustments are being made. Whenever the instrument is to be left, D must be closed; the column at A should be high enough to make the pressure at D outward, and thus avoid introduction of air. The meniscus in C should also be drawn down to a point below the reference mark, so that any soiling of the tube during long standing will not be at the reference mark.

2d. This portion of the experiment consists in measuring the pressure of the air when the bulb is at the temperature of the steam from water boiling under a measured atmospheric pressure. Having carefully removed the ice from around P , raise the bulb, remove the ice-box and stand, and replace them by the copper heater, containing about an inch depth of water and resting upon the gas-stove. Or if the heater and ice-box are combined, light the gas beneath the apparatus, and allow the ice to melt as the heating progresses. In placing the cover upon the heater, put the marks upon cover and heater together, as they fit loosely in this position. As soon as the water begins to boil, raise A about nine inches; then open D , and adjust the mercury to C as before. Read again the height of C and A , and note the thermometer, which now indicates t_2 . Read also the barometer and attached thermometer.

In careful measurements with the cathetometer, it is important that a certain sequence should be used in the readings of the barometer C , A , etc., in all cases. For this, inquire of the instructor.

Close D , and turn off the gas. Call H_2 the corrected barometer reading, and h_2 the corrected difference of level of $A - C$. Then $H_2 + h_2$ is the observed pressure of the air in the bulb. The volume of the air above C is now, as before, v ; the slight expansion of the glass from t_1° to t_2° being negligible in this correction, but its temperature is t_2 , so that its volume at t_1° would be $\frac{v}{1 + \alpha t_2}$. The volume of the bulb at 0° is V_0 ; but at T° (=the temperature of the steam) it is $V_0(1 + kT)$ when k is the coefficient of expansion of the glass ($=0.000026$). This volume of air reduced to 0° would, therefore, be $V_0 \frac{1 + kT}{1 + \alpha T}$. Thus the total volume of the air in the bulb and above C reduced to 0° would be $V_0 \frac{1 + kT}{1 + \alpha T} + \frac{v}{1 + \alpha T}$ under a pressure of $H_2 + h_2$.

Now, from Mariotte's Law, we know that for the same mass of gas, at a constant temperature, the product of the pressure by the volume is a constant. Hence, the product of the volume of the air reduced to 0° by its pressure in each of the above cases, since the total mass of the air is unchanged, should be the same, and we have, —

$$\left(V_0 + \frac{v}{1 + \alpha t_1} \right) (H_1 + h_1) = \left(V_0 \frac{1 + kT}{1 + \alpha T} + \frac{v}{1 + \alpha t_2} \right) (H_2 + h_2);$$

or, dividing both sides by V_0 ,

$$\left(1 + \frac{v}{V_0} \cdot \frac{1}{1 + \alpha t_1} \right) (H_1 + h_1) = \left(\frac{1 + kT}{1 + \alpha T} + \frac{v}{V_0} \cdot \frac{1}{1 + \alpha t_2} \right) (H_2 + h_2).$$

In this equation, the only unknown quantity is α , which may, therefore, be calculated. To facilitate the computation, the approximate value $\alpha = 0.003670$ may be substituted in the term $\frac{v}{V_0} \cdot \frac{1}{1 + \alpha t_1}$ and $\frac{v}{V_0} \cdot \frac{1}{1 + \alpha t_2}$, these being small

corrections only, and a value of α thus deduced. Should this value of α differ by 1 part in 500 from the assumed value, substitute it in place of the assumed value, in the two terms referred to, and recalculate α , thus obtaining a more correct value, one recalculation is ordinarily sufficient.

The temperature T of the steam may be ascertained as stated at p. 49 of these notes.

When the cathetometer is used, and the greatest attainable precision is sought, correction must be applied to this value of T for the change in barometric height by the variation in the force of gravity with the latitude and elevation above the sea.

To use this instrument as an air thermometer, it is necessary first to determine, once for all, α as just described; then immerse the bulb in any bath of which the temperature is to be measured; measure $A - C$ and the barometric height, and substituting H and h as thus found in above formula, compute the value of T .

3d. The value of V and v are to be obtained as follows if they are not required with a precision of more than 1 part in 50.

Obtain by measurements with a string, tape, or otherwise, as nearly as possible, the average circumference of the bulb. Allow about 0.5 mm. for thickness of the glass, and compute the capacity V in cc. When possible, it is, of course, best to determine V by weighing the bulb filled with water. To obtain v , turn D into the position 5; raise A somewhat above C ; open D slightly (position 3), so that the mercury may gradually flow into the capillary CE , and let it flow to the point E about where it was immersed in steam. The capillary is so small that a difference of some inches in this position will not introduce much error into v . A careful adjustment is necessary only in the most precise investigations. Then turn D into position 4, holding beneath the side-opening a small beaker to take the outflowing mercury. Draw off the mercury until the meniscus is as in the measurements, just at C . The volume of mercury drawn off must then be v . Weigh the mercury. Its volume will, of course, be $\frac{\text{wt.}}{\text{Sp. gr.}} = \frac{\text{wt. in grammes.}}{13.6}$.

Repeat this determination of v , and average the two results.

Care must be used in the computations of the results that the proper number of places of significant figures is used at all points in the work, so that the computation shall not vitiate the results. With the simplest form of apparatus used in the laboratory, results should be obtained within 0.5 per cent of the true value, which is 0.003670. If the cathetometer is used, and the air thoroughly dried, the results should be within 0.2 per cent. With imperfectly dried air, a considerably greater value of α is obtained, even up to one per cent too large.

When any considerable number of computations of either α or T are to be made, much saving of labor is effected by using the following simplified formula, and throwing the work into systematic form. Both α and T should be computed from the same formula.

$$\alpha T = \frac{H - h' + \frac{v}{V} \cdot H \cdot \frac{1}{1 + \gamma t_2}}{h' - \frac{v}{V} \cdot H \cdot \frac{1}{1 + \gamma t_2} - H \frac{k}{\alpha}}$$

where

$$\gamma = \alpha - k = 0.003644,$$

$$h = H_1 + h_1,$$

$$h' = h \left(1 + \frac{v}{V} \cdot \frac{1}{1 + \gamma t_1} \right),$$

$$H = H_2 + h_2.$$

Of course, when the greatest precision is sought, the values of k must be determined for the particular bulb and for all temperatures.

ADJUSTMENTS OF THE CATHETOMETER.

The following adjustments include the most important details in the use of the Staudinger Cathetometer. They are best made in the sequence here indicated, although in some cases this order is not essential. Thus adjustments 7 and 8 can be made at any time independently of others, and 7 requires frequent attention. Adjustments 1, 2, 3, and 8 should never be disturbed unless found to be imperfectly made. The student should note carefully the points which should be attended to at each use of the instrument, and be sure that they are never neglected, as injury to the instrument, or imperfect results would be sure to follow. Adjustments 1 and 2 once rightly made require no further attention, and are, from their nature, performed when the instrument is first used after being set up. Adjustment 3 requires attention only when the slide is seen to be slightly loose, or much too tight, but the bar must be carefully wiped and oiled before use on each separate day of use. In every motion of the slide up or down, the vernier must be turned away from the scale lest it wear a groove, and thus lessen the precision of reading. Adjustment 4 once made remains right unless disturbed in other manipulation. It must be occasionally tested. Adjustment 5 must be tested at least as often as at the beginning of each day's work, and 6 is made at the same time. Adjustment 7 must be made whenever the cross-wires are not in distinct focus. Adjustment 8 once properly made requires no further attention except at the beginning of any important work.

1. On the central steel shaft of the instrument are two conical bearings, one near its top, the other near its foot. The hollow brass sleeve carrying the rotating portions of the instrument has two corresponding conical surfaces within, and thus, when in place, settles down firmly upon the bearings of the steel shaft. These make excellent guides for the rotation, but, as will be readily seen, the friction is great. To obviate this and still retain the advantage of the guides, the steel axis is finished at the top as a flat cone. The apex of this is pressed upon by a steel plate, the amount of the pressure being regulated by means of a screw turning through a nut in the top of the brass sleeve in the line of the axis. By sufficiently turning this screw downward, the whole weight of the rotating parts may be borne by the plate, and the friction on the guides thus removed. The best effect is reached, however, by relieving the cones of but part of the weight, adjusting the screw so that the freedom of rotation is a little short of complete, a good bearing being thus insured.

2. The bar carrying the graduated scale, and upon which the telescope and its attachments receive their vertical motion, should be very nearly parallel to the vertical axis of rotation. The only error arising from want of such parallelism is the apparent lengthening of distances measured; for it is obvious that, were the scale tipped from the vertical through an angle α , a vertical distance, found by measurement with the inclined bar to be l , would, in reality, be $l \cos \alpha$. This adjustment is really made, at first approximately, by judgment by the eye alone. Then, after the later adjustments, this can be finally made with sufficient closeness by a carpenter's level or a plumb-bob.

3. The slide carrying the telescope and attachment must run easily on the bar without being loose. This adjustment, which is somewhat difficult, is made by means of a pair of screws on one side of the bar, which carry the portion of the slide bearing upon this edge of the bar. The least play of the slide allows change of level of telescope, and thus inconvenience and error. The bar should be thoroughly wiped and oiled on each day of use.

4. The adjustment of level so that it shall be parallel to the axis of the tele-

scope is necessary to facilitate subsequent adjustments. Place the level in its proper position on the telescope tube. Read the position of the bubble. Then turn the level end for end, and read again. If the level be in adjustment, the end of the bubble towards (say) the object glass of the telescope will each time be at the same distance from the centre line of the level. If these two readings are not the same, the adjusting screw provided at the end of the level must be turned until the condition is fulfilled.

The explanation is this. Let ef be the direction of the axis of the telescope, which may or may not be horizontal. Let cd be the direction of the surface at the middle point of the level when adjusted parallel of ef . In this case, the

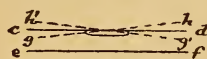


FIG. 34.

bubble will stand nearer the higher end. Now, on reversing cd , c will, of course, be brought to d , and d to c ; the inclination of the surface of the level will therefore be unchanged. Hence the end of the bubble will still stand at the same distance from the middle, and toward the higher end. If, however, the original position of cd had been gh , inclined to the proper line cd , then, on reversing, it would have moved to $g'h'$, and the bubble would have changed position.

By turning the telescope into parallelism with two of the levelling screws of the base, clamping the vertical axis, and then approximately levelling by means of these screws, this operation is facilitated.

5. Rendering the axis of the telescope accurately perpendicular to the vertical axis of the cathetometer.

First render the axis approximately vertical, by going through once with adjustment 6. Then read the position of the bubble. Turn the instrument about its vertical axis as nearly as possible 180° . If the telescope be perpendicular to this axis, the bubble, if not exactly in the middle of the tube, will shift to a second position symmetrical with the first with regard to the middle point of the level. In other words, if, in the first position, the reading of the end of the bubble towards the highest end of the tube be taken, then, after the 180° revolution, the reading of the end of the bubble towards the highest end of the telescope will be the same as before, but it will be the other end of both bubble and telescope. For, if mn represent the axis of the instrument, — not necessarily vertical as yet, — and cd the direction of the middle of the surface of the level (which has been rendered parallel to the axis of the telescope) when



FIG. 35.

perpendicular to mn ; and, if a be the position of the end of the bubble toward the high end of the telescope; then, as cd is perpendicular to the axis mn , if rotated 180° around that axis it will come into the same direction, but with c brought to d , and d to c . Hence the middle surface of the level will have the same direction, and the opposite end of the bubble will now come to the same distance from mn as before, but now a will be toward the opposite end d of the telescope. It will be readily seen, on inspection, that this will not hold true when cd is not perpendicular to mn .

Inasmuch, however, as it is difficult to turn cd through exactly 180° by the eye, and as mn cannot yet be rendered exactly vertical, it is evident that in the second position of cd a slight difference in position of a may arise, in virtue of these causes combined, even where cd is exactly perpendicular to mn . The adjustment by this method is, therefore, an extremely tedious one, and the following is to be preferred.

The whole instrument rests on three levelling screws, and by these it should be approximately levelled by performing 6 once. Turn the instrument around its

vertical axis until the telescope is in line with one pair, which may be called I. and II. of these screws. Lower the micrometer thumb-screw under the end of the telescope (in use this adjusting screw should always be at the eye end of the telescope), so that the round-headed screw shall rest on the middle of the movable steel plate provided for it. Adjust this screw until the middle of the bubble is at the middle of the level. Turn the instrument until the telescope stands over screws II. and III. Now turn screw III. until the bubble is again in the middle. Revolve the instrument, until the telescope stands over screws III. and I., and find how many divisions the bubble is away from its middle position. If it still remains in the middle, cd is perpendicular to mn . If the difference from the middle is p divisions, then the round-headed adjusting screw under the eye-piece is to be turned until this difference becomes $\frac{1}{2}p$, when the adjustment is approximately correct. If, however, one end of the bubble is concealed, or is very near one end of the tube, the necessary change in the adjusting screw may be considerably less than that corresponding to $\frac{1}{2}p$, owing to the rapid change of curvature of the tube of the level near the end. A repetition of the whole operation, beginning with the telescope over I. and II., levelling by II. until bubble comes to middle; turning to II. and III., and levelling by III.; then turning to III. and I., and reading the bubble again, will either show the former adjustment to be accurate or enable the observer to improve it. Two or three repetitions in this way will give a correct adjustment, and it is to be observed that, by this operation, the central axis mn incidentally becomes vertical, thus rendering 6 unnecessary. The reason for this halving of p will be readily seen on considering that if in the first position, I. to II., the line cd have the end c over I. and d over II., and if cd instead of being perpendicular to mn have the end d a little too high, then on adjusting II. until the bubble comes to the middle, the foot at II. is necessarily turned too low by a certain small amount. In the same way in the position II. to III., the foot at III. is turned below that at II. by an equal amount. So that when the position III. to I. is reached, the foot at I. is found higher than the foot at III. by the sum of these two equal quantities. Therefore, halving the inclination of the level in this point, which is approximately done by moving the bubble back to $\frac{1}{2}p$, will make the line cd nearly or quite perpendicular to mn . This adjustment must be tested on each different day of use of the cathetometer. Moreover, as the slide bar can never be perfectly true, the adjustment will not be exactly the same for all parts of the bar. This error is nearly eliminated by the use of the thumb-screw under the eye-piece, by means of which the telescope can and should be levelled at every observation, the thumb-screw being turned up to a bearing, and the plate turned out from under the round-headed screw before observations are begun.

6. Levelling to render the axis of rotation of the whole instrument vertical.

Bring the telescope and level into a position parallel to the line joining any two of the levelling screws, say I. and II., at the base of the instrument. Turn them until the bubble is at its middle point. Rotate the instrument until the telescope is over II. and III., and adjust III. until the bubble is in the middle. Then bring the telescope over III. and I., and the bubble should still be at its middle position. If this is not the case the operation should be carefully repeated. Should this fail to correct the difficulty, it shows that the preceding adjustment (5) has not been correctly made, or has been disturbed, and must be set right.

7. To focus the cross-wires by means of the eye-piece.

With the positive eye-piece, such as that in this telescope, the operation of focussing the cross-wires consists merely in drawing out or pushing in the eye-piece until the wires are perfectly distinct. The best test of the correctness of

this focus is that when the telescope is *carefully* focused on some distinct point, the wires do not appear to move over this point as the eye is moved from side to side of the eye-piece. This adjustment is an important one in all instruments using cross-wires. In making it, the eye should be repeatedly taken from the telescope for a moment's rest, so that no strain of the eye may cause a slightly faulty setting of the wires, which would produce in turn slight inaccuracy, and unnecessary fatigue in the after use of the instrument.

8. Centering the cross-wires.

This is necessary in order to bring into coincidence the optical and geometrical axes of the telescope. Without this adjustment an error would enter when the points, whose difference of level is to be measured, lie at different distances from the telescope.

Let the telescope rest in position on its support. Focus it on some minute and well-defined point, and direct the intersection of the cross-wires carefully upon that point. Rotate the telescope in its supports,—the vertical axis being clamped,—and observe carefully whether the intersection revolves about the fixed point or remains exactly upon it whatever the angular displacement of the tube. The latter will be the case when the wires are in adjustment. If they are found not to be so, they must be set right by repeated trials, the adjustment being effected by means of the screws near the eye-piece which hold the ring bearing the wires. This adjustment requires only an occasional test, and the cross-wires should never be disturbed except when out of adjustment.

MAGNETIC INCLINATION OR DIP.

Read carefully Kohlrausch, "Physical Measurements," pp. 129-132. It is also recommended that the student read Chap. xvii. of Glazebrook and Shaw's "Practical Physics."

Before taking any readings, remove the needle from the instrument and reverse its magnetism carefully, as directed at page 132 of Kohlrausch. Then study the distribution of magnetization in the needle by placing it on the board provided for the purpose, and beneath a sheet of paper, and sprinkling the paper with fine iron filings. If any "consequent points" or irregularities in magnetization appear, the remagnetization must be repeated in reverse direction. Care must be taken to use the same bar or horseshoe magnet each time in remagnetizing, and to give the same number of strokes, so that the magnetic moment will be as nearly as possible the same in all cases. Be sure to remove all particles of iron filings from the needle before replacing it on its bearings. Also, always replace both bar magnets in their box, in contact with the armatures and with opposite poles together, or if a horseshoe magnet is used put on its armature; and remove all magnets to a distance such that they shall not affect the needle, being careful not to disturb other electrical or magnetic work that may be going on near by. To study the difference in effect produced by the magnets when without and when with armature, sprinkle iron filings on a sheet of paper laid over the magnet in each case, and observe that, with the armature in place, the larger portion of the lines of force pass through the armature and disappear from surrounding space. To test whether the magnets are sufficiently far removed, read the dipping circle, then turn the box of magnets end for end and read again. This reversal should not affect the needle reading. Put all the apparatus together on leaving the experiment.

To set the plane of the mirror in the direction of the magnetic meridian, remove the needle to a distance from the instrument. Place the instrument on the cardboard base in such a position that the plane of the mirror passes through or is parallel to the line marked upon the board, and place a small compass on the board with its N. and S. line parallel also to this line, and turn the board upon the table until the line, and thus also the plane, of the mirror are N. and S. Level the instrument carefully, read the index on the horizontal circle, remove the compass to a suitable distance, and replace the needle on the circle.

In the instrument used in this experiment, the needle is supported on a steel pinion which rolls on agate plates. The friction is so great that without suitable precautions the readings may be several degrees in error. To eliminate the effect of friction, care should be taken before each reading to raise the needle off its bearings by the screw and lever provided for that purpose, and then to lower it again into place, thus insuring its proper placing. After the needle has come nearly or quite to rest, the base of the instrument must be lightly tapped (*e.g.*, with a pencil) until the needle settles into its final position of rest. The reading of both ends (the point just covering its reflection) of the needle should then be taken to 0.1° , and the needle again set swinging for a repetition. At least three settings should thus be made in each position. The jarring is intended to be sufficient to just lift the pivots from their bearings, and thus to nearly eliminate the effect of friction. Too forcible tapping will give the needle a throw of sufficient violence to disturb its position. If the jarring be carefully performed, — and this is the difficult part of the manipulation, — the readings will be quite accordant. A series of thirty readings gave an average deviation from the mean of less than 0.1° .

Three independent determinations of the angle of dip i are to be made, and the mean value of i computed, using Kohlrausch's formula I. One set of results is also to be computed by II. and III.

It is important to avoid disturbance of the needle by air currents due to the breath, and to see that the instrument is at a suitable distance from masses of iron. Note that the circle is numbered from 0° to 180° , from the bottom to the top in both directions. Hence, in order that the angles should correspond to the suppositions upon which Kohlrausch's formulæ are deduced, a proper correction must be applied to the readings. Find this by inspection.

Assuming the horizontal component of the earth's magnetism at this place to be 0.1717 cgs., what would be the total intensity of the field as deduced from the mean result of the foregoing measurement?

USE OF GALVANOSCOPES.

In the measurements of many kinds of electrical magnitudes, and particularly in general practice, there is a large number of methods in which a galvanoscope of simple and cheap construction may be employed, a galvanometer of accurate design and workmanship being unnecessary. Besides this obvious advantage, these methods possess, more or less completely, others still more important, viz.: 1st, elimination of all constants of the instrument, and thus of many instrumental errors; 2d, increased simplicity of procedure and computation, and greater uniformity of conditions in various steps of the measurement; 3d, possibility of use of much more sensitive instruments than can often be produced when it is necessary (as, *e.g.*, in the tangent galvanometer. See p. —) to adhere to a fixed proportionment of parts.

Galvanoscopes of great sensitiveness in proportion to their resistance may be made, by winding the coils close about a short suspended needle. The application of such will be indicated in the subsequent notes on Resistance by Substitution, E.M.F. and B., Resistance of Galvanometers, Wheatstone's Bridge, etc.

There are two chief functions of galvanoscopes: 1st, to indicate when no current is passing in a circuit; 2d, to indicate, with two or more different arrangements of the apparatus, the passage of equal currents in a circuit. The first case requires simply that the galvanoscope be sufficiently sensitive.

Neutralizing Magnet.—If the form of coils and needle be not such as to give sufficient sensitiveness, this may be very greatly increased by reducing, by a neutralizing magnet, the intensity of the magnetic field about the needle. This simple process, often overlooked, may render great precision attainable with a crude galvanoscope. It is habitually used in the Thomson galvanometer and similar scientific instruments.

Procedure.—A bar or other strong magnet is placed in such a position relatively to the needle of the galvanoscope that its field at the needle should be of the same direction as that of the earth's field at that point, but of opposite sign. The resultant field is thus the difference of the two fields, and by varying the distance of the magnet, this difference may be made as small as desired. The deflection of the needle by a given current is inversely as some function of the intensity of this field, so that the sensitiveness increases with the diminution of the intensity of the field. The usual and best position of the galvanoscope (see notes on Thomson galvanometer) is with its needle normally in the magnetic meridian, and the neutralizing magnet is then placed with its axis in a north and south horizontal line, either in the prolongation of the axis of the

needle, or vertically above or below it. When it is necessary to render the field very feeble, this may be most conveniently done by adjusting the position of one magnet until the reduction is slightly less than that desired, and then completing by another magnet so much more distant that a comparatively large movement produces little difference in the field at the needle.

The reverse of this process is also sometimes useful, *i.e.*, increasing the intensity of the field, and thus diminishing the sensitiveness of the galvanoscope when too sensitive for a given purpose.

Discussion.—The needle would be astatic if the field were zero. The only special condition in this and the “compensation” process is that the needle shall be of so short length that portion of the field through which it moves shall be tolerably uniform; otherwise, there may be points of unstable equilibrium in its motion.

The second case, *i.e.*, the adjustment of equal currents, may be treated in two ways, as follows:—

a. Equal Deflection Method.—The same current will of course be indicated by equal deflections on the galvanoscope. The fractional precision of this adjustment depends on that with which the reading can be taken by the index on the graduated circle, by the spot of light, on the scale, by the telescope, or by whatever means is employed, and on the magnitude of the total deflection. With the graduated circle (in degrees) and index this fractional precision can seldom reach 0.1 per cent, and is often less than 1 per cent. With the mirror and spot of light, a long beam of light must be used to get 0.1 per cent, since the angle of deflection must be small. With the telescope 0.1 per cent is more readily attainable than with the spot of light. In ordinary work the index and circle is the most convenient, and often the only available means of reading. The neutralizing magnet may be employed in this method to extend its range to either greater or smaller currents, but it is not generally convenient. The chief difficulty with the equal deflection method in general use is, that in order to have the index at a proper point on the scale, the limits of strength of current which may be used are quite narrow, which is not true of the following method.

b. Compensation Method. Procedure.—While a current is passing through the galvanoscope, a magnet is approached to it in such a position that its field at the needle is sensibly equal in intensity, and opposite in direction, to that due to the current. The needle is thus brought back to its initial position, and is in a resultant field sensibly the same as that in which it was before any current passed through the coils. If, then, this compensating magnet and the galvanoscope remain undisturbed (and also, of course, the direction of the original magnetic field at the needle), the current passing through the galvanoscope will be again the same only when the needle is at this initial position (usually, of course, zero), and thus it is possible to determine when the same current is again established. It is unnecessary in this method that the galvanoscope needle should be originally adjusted precisely to zero, or that the compensation should bring the needle exactly back to its initial point. The only requisite is that the reading of one end of the needle when approximately compensated should be *exactly* noted, and the same deflection of the same end of the needle *exactly* reproduced when it is desired to reproduce the current.

Discussion.—Since the conditions of the galvanoscope are precisely the same at both observations, all instrumental errors (except any such irregular one as friction) are wholly eliminated. The gain in precision of this method results from the possibility of utilizing by it the full sensitiveness of any galvanoscope with all strengths of current within the limit prescribed by possible

overheating of the coils. With the equal deflection method, the limiting strengths of current which can be used, with a precision of 0.5 per cent, when deflections are read to 0.1° , are such as will give deflections between about 20° and about 70° , allowing a range of currents of 1:5 at most (*i.e.*, for tangent compass). This range can be extended to larger or smaller currents by the proper use of the neutralizing magnet, but in the latter case at the expense of promptness of motion, and stability of adjustment of the needle. The precision of the method is the same with or without the neutralizing magnets. But by the use of the same galvanoscope and the compensating magnet (or magnets), currents from the above-named lower limit of 20° up to the largest which the coils can carry without injury, can be measured with a precision increasing with the current. For when the needle is brought to zero by the compensating magnets, it will be in such a field that the change of current necessary to produce 0.1° deflection will be the same as to deflect the needle from 0° to 0.1° if no initial current were passing. This, in a tangent galvanometer, would correspond to an error of about 0.5 per cent for a current which would deflect the needle to 20° , about 0.2 per cent for 45° , about 0.1 for 60° , and so on. The method is thus capable of determining a current which would give a deflection above the greatest with which the equal deflection method could deal, and of making this determination with greater fractional precision than is possible at any deflection on the same galvanoscope, by the equal deflection method. The precision increases with the magnitude of the deflection which the given current produces on the galvanoscope, and it is, therefore, desirable to have the coils so wound as to have the greatest possible sensitiveness with the largest wire which can be advantageously used. The strength or number of compensating magnets to be used must, of course, increase with the sensitiveness of the galvanoscope and strength of current.

The neutralizing magnets may, of course, be employed to still further extend the sensitiveness of the arrangement. Also it is possible to make the compensation by an auxiliary coil with a separate current, instead of by magnets.

MEASUREMENT OF RESISTANCE BY SUBSTITUTION.

The simplest method for the measurement of the electrical resistance of conductors is based on a direct application of Ohm's Law. Suppose a closed circuit consisting of a constant battery, a resistance to be measured, and a galvanoscope. The latter will show a deflection corresponding to the current passing. If, now, the resistance to be measured be removed, this current and the deflection will increase, and the insertion of a resistance precisely equal to the one removed will be necessary to bring the current again to the same strength, and thus the needle to the same reading, as in the first instance. If this latter resistance be inserted by means of known resistance coils, then their amount will give the measure of the resistance sought. This method is called the measurement of resistance by substitution. The process just described is known as substitution by "equal deflections," since the current is always brought to the same amount by reproducing upon the galvanoscope the original deflection. A second process of using the substitution method is by "compensation."

Read the notes on "Use of Galvanoscopes," page 62, and, in the measurements of this experiment, follow the directions there given for both "equal deflection" and "compensation" process, using the "neutralizing magnet" if necessary.

That the galvanoscope should be sufficiently sensitive to a given percentage change in the resistance to be measured requires that this resistance should be a considerable fraction of the whole resistance, and thus that the galvanoscope and battery resistance should not be excessive. The battery resistance should be as small as is consistent with the necessary electromotive force, and the plate surface should be large, to diminish the effect of polarization. The battery employed must be constant. The electromotive force should not exceed that which, through the galvanoscope alone, or through the resistance to be measured alone, would produce sufficient heating effect to change these resistances by an amount corresponding to the precision of measurement desired. The ratio of the galvanoscope to the exterior resistance must depend on the sensitiveness of the galvanoscope in any individual case, being less, of course, for the same precision as the sensitiveness is greater. When absolute resistances are desired, the temperature must be taken into consideration. (See notes on Temperature Correction in Resistance Measurements.) The method is best adapted to rapid measurements of moderate resistances, with a precision of one per cent to one-tenth per cent, and its main practical advantage is that, with the exception of the box of resistance coils, only crude apparatus is needed. A Thompson reflecting galvanometer more or less shunted may be used, but any short needle sensitive galvanoscope will answer most requirements.

Kempe's "Handbook of Electrical Testing," Chaps. i. to iii. inclusive, may advantageously be read in this connection. Record in note-book diagrams of the component and resultant forces producing equilibrium of the deflected needle, in the conditions of no compensation, of over, under, and exact compensation, without the use of a neutralizing magnet; and also of exact compensation with the use of the neutralizing magnet.

Measure the resistance of the coil *C*, first by the deflection method, then by the compensation method. Then measure each of the other coils *A* and *B* by the compensation method. In each case make at least three independent measurements, and compute the average deviation of each series. Measure also *C* and *A* in parallel circuit, and compute from the former measurements what this should be. (Pickering's "Physical Manipulation," ii. 259.) Also measure *A*, *B*, and *C* in parallel circuit, and compare the computed and observed resistance as before. In using the galvanometer, care must be taken to see that the needle swings clear. The galvanometer may contain two or more sets of coils, one of which is short and of few turns, and is connected between the binding posts marked *S*; the other is longer, *i.e.*, of more turns, and is marked *L*. Use in any case the coil which produces a suitable deflection with the given resistance in circuit. It will be found that with the smaller resistances, the long coil may give a greater deflection than can be neutralized by the use of even two ordinary bar magnets. In such case the short coil should be used. In using the bar magnets, take pains to disturb the work of others as little as possible. Leave the magnets in their box, with opposite poles in contact with the same armature. If a Daniell battery is used, remove the porous cup and zinc to the extra jar containing the zinc sulphate solution, at the close of the work.

DIFFERENTIAL GALVANOMETER.

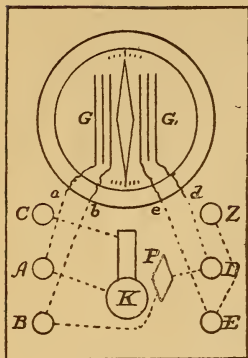


FIG. 36.

The differential galvanometer consists of two coils of wire of equal resistance, and so placed that when the same current is passed through both at the same time and in opposite directions, no deviation of the magnetic needle suspended within them is produced. The wires composing the two coils are usually wound side by side on the same spool, or are twisted together, and then wound as one wire. The Clark differential galvanometer is arranged as shown in Fig. 36. G and G_1 are the two coils, so connected that a current entering at A and going out at B will pass through G and cause the north end of the needle to deflect (e.g., to the west). A current entering at D and going out at E would then pass through G_1 , and deflect the north end to the east. Thus, if the negative (copper, carbon, etc.) plate of the battery be connected to C , and the positive plate to Z , upon pressing down the key K , the current will flow from C through K to A , thence through G to B , through the diagonal wire and plug P to D , thence through G_1 to E , Z , and back to the battery. If the plug is out, or the key open, the current will not pass. The plug is withdrawn when it is desired to use the coils in separate circuits. Set the galvanometer up with the plane of the coils in a north and south direction. Try each coil separately, and the two in series, noting the direction of the current and of the deflection in each case, and whether any deflection, and how much, occurs when the current passes from C to Z . Unless this corresponds to a sufficiently small fraction of the whole current passing, and unless the resistance $AGB = DG_1E$ very nearly, the resistance of x , as determined between A and B , will be different from that found when x is between D and E . If this difference is quite small, the mean results will be close enough; but if it is not small, the galvanometer requires readjustment.

The simplest method of measuring a resistance with the differential galvanometer is to connect between the binding screws A and B , a box, R_1 , of resistance coils, and between D and E the resistance R_2 to be measured, or *vice versa*. Connect the — plate of the battery with C , and the + with Z . When the circuit is closed by pressing the key, the plug being in place, the current passes to A , where it divides according to the law of shunts; part passing through G , part through R_1 , in the inverse ratio of the resistances. These parts reunite at B , flow to D , there are divided again between G_1 and R_2 , reunite at E , and flow out at Z . Thus, since the resistance of G is exactly equal to that of G_1 , the same fraction of the whole current will flow through each only when $R_1 = R_2$. Hence to measure R_2 the coils at R_1 are adjusted until no deflection occurs upon pressing down the key, when the resistance of the coils is the desired resistance. Ordinarily it will be impossible to obtain from the resistance box such an amount as to just equal R_2 ; in such case the deflection with the resistance just too large, and with the one next smaller, may be noted, and by interpolation the fraction approximately found.

Find with the galvanometer the resistance of the wire on the spools I and II . successively. Then connect them side by side to form a “derived,” “parallel,” or “shunted” circuit, and measure the combined resistance. Compute this also by the law for shunts from the previous measurements, (Pickering’s

"Physical Manipulation," vol. ii. p. 259,) and compare the observed and computed results.

The differential galvanometer is convenient for many purposes. A simple way of finding approximately the resistance of a battery with it is as follows. Connect two (or any even number) of similar cells in opposition. Such a combination will ordinarily produce a slight current, as the electromotive force of one cell will seldom be exactly equal to that of another. This will soon be partly overcome if they are allowed to remain on a closed circuit for a few moments. It may also be partially eliminated in other ways, but will be a source of less error than that resulting from the polarization produced by the current from the main battery connected to C and Z . To measure $2B$, the resistance of the pair of cells, connect the like plates to D and E , and adjust R as nearly as possible, leaving the key down for only a moment at a time. It will be noticed usually that upon connecting to D and E , the needle will be deflected by the current from the pair of opposed cells. The amount of the deflection will change from time to time, and will be either reversed or increased according to its direction after the key has been closed for a few moments. The change is due to the reverse current sent out by the polarization caused by the current from the other battery.

Measure the resistance of the cells provided for the purpose, and note whatever changes in current and other effects of polarization you may be able to observe. This measurement will be found almost impracticable with cells like the Leclanché, but illustrates the difficulties inherent in the measurement of any liquid resistances, and of the resistance of single liquid batteries. The method is quite readily applicable to constant cells.

Differential galvanometers are often made with the wires coming out to three binding posts only. In such an instrument the coils G and G_1 would be the same as in Fig. 36, but with all the connections beyond *deba* removed. The end a would then be brought to one binding post, a , Fig. 37, the end d to another, c , and the two ends b and e , joined together, would be brought to a middle post b . The outside connections for a resistance measurement with such a galvanometer might be made, of course, to correspond with those of the Clark Differential, but they would be generally made as shown in Fig. 37. If such a galvanometer is provided, first test the galvanometer by connecting f direct to a and h to d . Then measure at x each of the coils already measured. Interchange R and x , and repeat both. It is convenient to have the key between either b or g and the battery.

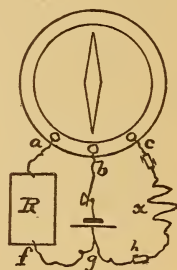


FIG. 37.

SLIDE WIRE BRIDGE, OR B. A. DIVIDED METRE BRIDGE.

Read Kempe's "Handbook of Electrical Testing," pp. 191-204.

Determine n_1 and n_2 (see § 212), using the coils marked A and B , whose resistances are stamped upon them.

Determine m_1 and m_2 (see § 214), using successively for m_1 and m_2 the coils marked C and D . Use at a the coil marked E (resistance is stamped on it), and at x the coil B .

Then using C and D at m_1 and m_2 , measure F , using G as a standard (resis. stamped upon it). Also measure A , using B as a standard. (See last formula in § 214.) Then remove m_1 and m_2 , and repeat these measurements with straps in at these points (formula in § 212). Note that the bridge readings for which Kempe's formulæ are arranged are in mm. In each measurement take at least five independent settings on the wire, record each, and find average deviation from mean. This quantity divided by the length of the shortest side of the wire (including the value of m on that side) will give an approximate measure of the precision of measurement, but the errors entering into the measurement of n_1 , n_2 , m_1 and m_2 , will, of course, affect any subsequent work involving the use of these quantities.

In the present experiment, various coils of wire made up for the purpose are used for the sake of convenience. In general, of course, adjustable resistance boxes might be employed, and the resistances varied to suit the demands of the "Best Conditions" (§§ 215 *et seq.*). When these conditions are carefully fulfilled, this instrument becomes capable of a high degree of precision, and, for some purposes, is very convenient. The principle of the divided wire is of very ready application for a variety of purposes in measurements requiring only moderate precision. For many purposes a galvanoscope of low resistance and rendered sufficiently sensitive by a neutralizing magnet (see notes on Use of Galvanoscopes) may be employed, as in the apparatus provided with this experiment.

WHEATSTONE'S BRIDGE.

The principle and theory of the bridge may be found in Pickering's "Phys. Manip." ii. 260, Ganots' "Physics," § 955, and must be read before beginning the experiment. In the diagram in Fig. 38 let M , N , X , and P , represent four resistances, then the points B and C will have the same potential only when these four resistances fulfil the proportion $\frac{M}{N} = \frac{x}{P}$, and in this case no current will flow through the galvanometer G when the circuit is closed. If M , N , and P are then known resistances, the resistance of x will be $x = \frac{M}{N} \cdot P$. The

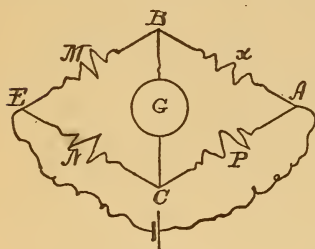


Fig. 38.

various actual forms of the bridge differ greatly from this diagram. The following directions apply, except as to details of the apparatus, to the use of any bridge. Record the number of the bridge used.

The Bridge.—The plugs between A and B and between B and C , Fig. 39, are supposed to be removed in all that follows, so that there is no connection between these points. The resistances corresponding to M , N , and P , in the resistance box LD , are in the lines EB , EFC , and $CDHLA$ respectively. They are in the form of double wound coils of wire within the box, each coil having ends attached to the brass block on either side of the corresponding plug. The resistances in ohms are stamped on the top opposite the plugs. (Details may be found in Jenkin's "Electricity and Magnetism," Kempe's "Handbook of Electrical Testing," etc.) Thus when any plug is removed, the current of electricity must pass through the corresponding coil in order to get from the brass block on one side of this plug to the next beyond the plug, and there will be thus interposed a known resistance. If the plug is firmly in place, its resistance is so

nearly zero as to be negligible. The coils in the line $ALHD$ are graded like the weights in a box of metric weights, and the insertion of resistances by the removal of plugs corresponds exactly to and must be proceeded with in exactly the same methodical way as the putting of weights into the scale pan of a balance. (See "Method of Weighing," p. 7.) The spot of light reflected from the galvanometer mirror upon the scale plays precisely the same part as the index or pointer of the balance. The lines of resistances M and N correspond,

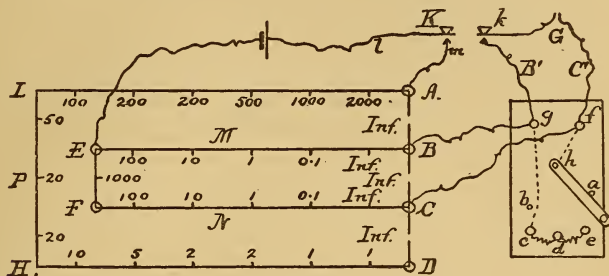


FIG. 39.

as will be seen on further inspection, to the arms of the beam of the balance; but they differ in this respect, that while the arms of the balance (except the steelyard, to which this instrument is even more closely analogous) are fixed in relative length and are generally equal, the bridge arms M and N may have several ratios. The use of the bridge with various ratios of $M:N$ is thus analogous to weighing in a balance in which the arms can be so altered in relative length as to make 1 gramme in (*e.g.*) the right pan counterbalance 10,000, 1000, 100, 10, 1, 0.1, 0.01, 0.001 or 0.0001 grammes (see numbers on M and N in Fig. 37) at will, in the left pan, thus varying the sensitiveness and range of the balance enormously at will.

Galvanometer.—A description of the galvanometer ordinarily used with the bridge (Thompson reflecting galvanometer) may be found in Jenkin, Kempe, and most text-books on electricity or physics. The proper adjustment of the instrument will be made and explained by the instructor at the request of the student. The two terminals of the galvanometer G are connected to the points B and C respectively of the bridge box. This connection is not direct, however, but through a key and a shunt.

The Shunt is an instrument for diverting more or less of the current passing from C to B , so that only a suitable portion shall go through the galvanometer itself. This prevents sending an excessive amount through the galvanometer, whereby it would often be injured and the procedure be always rendered unnecessarily slow. It would correspond to some attachment to a balance, whereby its sensitiveness might be modified at will. The construction of the shunt will be understood from reference to the figure and the instrument. The current coming from the bridge by the wire C will tend to pass along fh to the switch ha ; but if this be at the pin a , there will be no further circuit in this direction, and the whole current will therefore necessarily go through fC' to the galvanometer, then back through $kB'gB$ to the bridge, thus all going through the galvanometer, in which case that instrument will be at its full sensitiveness, — a condition in which it is to be used only when the final measurements are to be made and a less sensitiveness has been found insufficient. If the switch be turned to rest upon the pin e , then a part only of the current will pass through G , the remainder through the resistance coils joining e , d , and c , and thence to

g and *B*. If the switch is on *d*, only the coil *dc* will be in circuit; if at *e*, there is only the very small resistance of the switch and connections, so that almost the whole of the circuit coming in at *C* would now be directed through the path *fhcg* to *B*, and thus almost none go through the galvanometer. The switch should be in this position when the apparatus is left at the close of the work. The resistance of the coil between *d* and *c* is made equal to $\frac{1}{999}$ of the galvanometer resistance, so that by the law of shunts, when the switch is at *d*, 0.999 of the current passes through *fhdcg* and only 0.001 through the galvanometer. Similarly the resistance between *e* and *c* is $\frac{1}{100}$ of that of the galvanometer, so that with the switch on *e*, 0.1 of the current only passes through *G*. The switch should be put upon *d* at the outset, and always when any change in the resistances is made which is liable to cause any considerable change in the current flowing through *G*. The switch is changed to *e* only when sufficient deflection of the spot of light is not obtained with it on *d*, and to *a* only when the deflection is too small with it on *e*.

Keys are necessary, as at *K* and *k*, in both battery and galvanometer circuits, and it is essential that *K*, the key in the battery circuit, should be closed before *k*, and kept closed until after *k* is again opened. This allows the induced currents accompanying the beginning of the currents in the bridge box, unknown resistance, and connections to pass away before the galvanometer circuit is closed; and the breaking of the galvanometer circuit before the battery circuit prevents corresponding induced currents accompanying the cessation of the battery current from affecting the galvanometer. To accomplish this result easily, the keys are usually combined on a single base. This double key consists of an upper leaf of metal connected with one battery wire *e*, a second similar leaf connected with the continuation of this wire *m*. There is next a third leaf, similarly connected to one of the galvanometer wires, and below this a pin connected with the continuation of this wire. The leaves are all insulated from each other. On pressing down the upper leaf, it makes contact with the second, and thus closes the battery circuit. Pressing down still further causes the third leaf to make contact with the pin beneath it, thus closing the galvanometer circuit. On relieving the pressure gradually, the galvanometer circuit is first broken, then the battery circuit.

Procedure.—See that the connections are rightly made throughout, that all binding screws are firmly (not violently) turned up, and that *all* the plugs are firmly (not very forcibly) twisted into place. Ask the instructor to see that the galvanometer is properly adjusted, and to explain any matters not clear after a careful reading of *the whole* of these notes and references. Connect one end of the coil marked *B* under the upper nut of the post *A*, Fig. 37, the other end under the part at *B*. Draw the plugs between *A* and *B* and between *B* and *C*. When these are drawn, the connection, formerly made by the plugs, is wholly broken. These are called “infinity plugs,” as they serve to interpose an infinite resistance. Draw the 100-ohm plug in the line *EB*, and the 100-ohm plug on the line *FC*, thus making $M=100=N$, so that $M/N=100/100=1$, the arrangement thus becoming analogous to an equal-arm balance. Thus from the proportion $M:N=x:P$, it appears that for each ohm in *x*, one ohm must be drawn in *P*. Place the switch of the shunt on *d* (the one-thousandth shunt). Draw the 1000-ohm plug in *AL*, putting the loose plug in the hole provided for it in the middle of the adjoining block. Watch the spot of light on the screen; press the key for an instant only (but not violently), letting it up immediately, and note the direction in which the spot moves—say to the right. This resistance is probably too much. Replace *firmly* the 1000 plug, and try next the 1-ohm coil in the same way—suppose the spot to go to the left. Then 1 is too small; and the spot

goes to the right when the resistance unplugged is too large, to the left when too small. Replace the 1-ohm plug, and try the 500, 200, etc., successively, exactly as weights in a balance (Notes, p. 7) until it is found, *e.g.*, that 11 ohms is too large and 10 is too small. Record the ratio used and the resistance found. It is possible, by the deflection readings described later, to get nearer than 1 ohm with the present equality balance, but in this instance it is not necessary.

If it is found during this measurement that the deflections of the spot of light become less than about 1 cm. when the one-thousandth shunt is used, the switch must be shifted to the tenth shunt, and then, if necessary, to the stop *a*, so that the full current passes through the galvanometer. Care must be taken always to return the switch to the thousandth shunt when any considerable change is made in the plugs. These directions in regard to the use of the shunt apply equally to all the subsequent work with the bridge. It will be seen that this variation of the shunt corresponds to a change of sensitiveness of a balance, arrangements for which do not exist in any of the usual forms of balance.

The next step towards obtaining a more precise measurement of the resistance is to obtain a balance with a different ratio of the bridge resistances *M* and *N*. Having returned all the plugs except the infinity plugs firmly to place, take out the 1000-ohm plug between *E* and *F* and the 100-ohm plug between *E* and *B*. The ratio is now *M* : *N* = 100 : 1000, or 1 : 10. Hence for a balance *P* must be made = 10 *x*. The plugs in *A*, *L*, *H*, *D* must therefore be drawn systematically, starting with 110 ohms, using suitable shunts, until it is found, *e.g.*, that 108 ohms is too much, and 107 ohms is too little. The resistance of *x* will then be between $\frac{108}{10} = 10.8$ ohms and $\frac{107}{10} = 10.7$ ohms. 110 ohms is used at the start

because the previous measurement showed that 11 ohms was too much, and 10 ohms too little. It would be useless therefore to start with more than $10 \times 11 = 110$. And the object in starting with the resistance known to be too large is simply as a check to see that the spot goes to the right as before, with too large resistance unplugged. If it fails to do so, then either the plugs have not all been properly in place (a frequent occurrence with beginners) or the coils are not correctly adjusted to the amounts marked upon them (analogous to errors in adjustment of set of weights). The latter error can be corrected only by a table of corrections which sometimes accompanies the instrument. The next place of significant figures may now be obtained in two ways, both of which are to be tried. First, by *deflections*. This method must generally be employed to get the last place of significant figures. Bring the spot to the zero of the scale, when the key is up, by the bar magnet on the table, or by the neutralizing magnet upon the galvanometer if necessary. This and the following manipulations of the spot will be difficult when jarring of the building or magnetic disturbance keeps the needle in irregular swinging; in such case the student must get the best readings obtainable without spending an excessive amount of time. With *P* = 107 ohms and the ratio of *M* : *N* = 100 : 1000, and suitable shunt, press the key down, and hold it down, until the spot comes to its position of rest. Record the position of the spot, *e.g.*, 2.5 div. to left. Make *P* = 108, and record the position of the spot, *e.g.*, 6.3 div. to right. The spot therefore moves $2.5 + 6.3 = 8.8$ div. for 1 ohm change in *P* under these conditions (so that 8.8 corresponds to the sensitiveness of the balance (Notes, p. 11)). If therefore it were possible to add $\frac{2.5}{8.8}$ ohm = 0.28 or 0.3 ohm to *P* when *P* is 107. ohms, the spot would be brought up to the zero, *i.e.*, would remain undeflected when the key was pressed, and the balance would be complete. Thus the result would be

$x = \frac{M}{N} P = \frac{100}{1000} \times 107.3 = 10.73$ ohms. This process is exactly the analogue of "Weighing by Swing of Balance," p. 10, and may be carried out in exactly the same manner when the galvanometer is upon a sufficiently steady foundation. Second, make $M=10$ and $N=1000$, or $M=1$ and $N=100$. Either ratio is about equally good, and that usually preferable which produces the largest deflection on the galvanometer in the following work. Make $P=1100$, and then adjust until it is found, *e.g.*, that 1073 is nearly right. Then $x = \frac{10}{1000} \times 1073$, or $\frac{1}{100} \times 1073 = 10.73$. By the deflection method, another place of significant figures might now be found, using a sufficiently sensitive adjustment of the galvanometer or increased battery power, but with the apparatus used in this experiment, and in any case without introducing correction for the change of resistance of the wires with change of temperature, it is useless to carry the result to more than four places of significant figures.

Measure next the resistance of the coil marked A , obtaining as many places of significant figures as possible. Measure to four places the resistance of A and B connected in series, and compare with the resistance computed from the previous measurements; also measure them when connected in parallel circuit, and compare with the resistance as computed by law of shunts. Measure any other resistance provided with the apparatus.

The most sensitive arrangement of the bridge is with the galvanometer connecting the junction of the two highest resistances of the bridge with the two lowest, if G has a higher resistance than the battery, which is the usual case. Thus when M and x are greater than N and P , and G greater in resistance than the battery, G should be connected as in Fig. 37. The sensitiveness is ordinarily greater when M and N are as large as possible consistent with obtaining the desired ratio; for this reason it is undesirable to use the ratio of 0.1:0.1, 1.:1., 1.:0.1, etc. It will, of course, be readily understood that, apart from this reason, the precision would be greater with larger coils in M and N , because these are more likely to be adjusted correctly to the fourth decimal place, and also the slight increase of resistance due to loose plugs in the arms M or N would be a less fraction of the total resistance of these arms. It is also evident that the slight increase in the ratio obtainable by making, *e.g.*, $N=1100$ when $N=10$ over making $N=1000$ when $N=10$, would be much overbalanced by the inconvenience of the ratio $\frac{1100}{10} = \frac{110}{1}$ as compared with $\frac{1000}{10} = \frac{100}{1}$. The infinity plugs at I and J are of service when the box is applied to some other uses, and for the present use may be a source of error.

With a bridge containing the coils shown in Fig. 39, show what would be the largest and what the smallest resistance which could be measured to four places of significant figures, presupposing sufficient sensitiveness of the galvanometer. Note that by changing the battery wire from E to F , the 1000 coil can be thrown into M when so needed to obtain ratios suitable for measuring large resistances. Give two demonstrations of the formula for the bridge, one by the graphical method, one by the use of Kirchhoff's Laws.

TEMPERATURE CORRECTION IN RESISTANCE MEASUREMENTS.

The temperature of the coils of the resistance box should be measured by a thermometer having its bulb inside the resistance box. Precise work cannot be done where the temperature is rapidly changing; indeed, absolute measurements with precision as great as 0.1 per cent require numerous precautions. Unless the temperature is nearly constant, it is impossible to ascertain the average temperature of the wire in the coils with sufficient closeness. The temperature of the wire whose resistance is to be measured must also be taken with a precision dependent upon the accuracy sought. Since the resistance of most pure metals increases about 0.4 per cent per degree centigrade rise in temperature, it is obvious that if results to one per cent are desired, no measurements should be taken without observations of the temperatures at the time.

Suppose that the resistance W of a piece of wire at an observed temperature, T° , is measured by a bridge (or other resistance coils) whose observed temperature is t° ; and suppose that these coils are standard at τ° (usually marked on the box). It is required to find, first, the resistance W_1 of W at T° in terms of the standard ohm, *i.e.*, of these coils at τ° ; and, second, the resistance W_2 of W at some chosen temperature, T_2° , of reference.

Let β_t be the mean coefficient of change of resistance with temperature for the measuring coils between 0° and t° , then $\beta_t = a + bt$, in which the values of a and b , as obtained by Matthiessen [Jenkin's "Electricity and Magnetism," p. 253], are:—

	a	b
Most pure metals	+ 0.003824	+ 0.00000126
German silver	+ 0.0004433	— 0.000000152

1st. Note that the resistance of the bridge coils at t° would be reduced at 0° in the proportion of $1 : (1 + \beta_t t)$. The fixed wire resistance W , measured as W apparent units when the bridge was at t° , would therefore have been found as $W(1 + \beta_t t)$ units had the bridge been at 0° . And similarly had the bridge temperature been τ° when the measurements were made, the resistance would have been found as

$$W_1 = W \frac{1 + \beta_t t}{1 + \beta_\tau \tau} \text{ standard ohms.}$$

2d. Let δ be the mean temperature coefficient for the measured wire (found as β was); then this wire would possess at 0° a resistance in standard ohms,

$$\frac{W_1}{1 + \delta_\tau T'}$$

At any other temperature, T_2° its resistance in standard units would be

$$W_2 = W_1 \frac{1 + \delta_{T_2} T_2}{1 + \delta_\tau T'} = W \frac{1 + \beta_t t}{1 + \beta_\tau \tau} \cdot \frac{1 + \delta_{T_2} T_2}{1 + \delta_\tau T'}$$

These expressions become simplified when $T_2 = 0^\circ$, as $\delta_{T_2} T_2$ disappears. When both wire and coils are of same material, so that $\beta = \delta$, and when $t = T$ and $T_2 = 0$, W_2 becomes

$$W_2 = \frac{W}{1 + \beta_\tau \tau}$$

If t and τ are but 2° or 3° apart,

$$\frac{1 + \beta_t t}{1 + \beta_\tau \tau} = 1 + \gamma_\tau (t - \tau) \text{ approx.,}$$

where $\gamma_r = a + 2 \beta \tau$ = the "true coefficient" of change of resistance of the wire with change of temperature. Most resistance coils are made of German silver.

If the work is of precision not much exceeding 1 per cent, the change of resistance may be allowed for by adding or subtracting 0.4 per cent per degree for most pure metals, and 0.04 per cent for German silver.

Prepared tables of resistances of copper and other substances at various temperatures are given in most electrical handbooks (Wigan's "Electrician's Pocket Book," p. 165), and are convenient for ordinary purposes.

SPECIFIC RESISTANCE.

The experiment on Wheatstone's Bridge must have been performed before this is begun. Read also the following references before beginning work: Kempe, "Handbook of Electrical Measurements" (3d ed.), pp. 166 to 172 inclusive, stating in your record the conclusions arrived at in §§ 196 and 197; pp. 31 to 43, but do not disturb the adjustment of the galvanometer, as practice in setting this up is given in connection with the work in measurement of electrostatic capacity; pp. 10 to 15; Jenkin, "Electricity and Magnetism" (7th ed.), chap. xvi. §§ 14 to 17; Wigan, "Electrician's Pocket Book" (2d year's ed.), pp. 162 to 165.

In using resistance coils, great care should always be exercised to avoid sending excessive currents through them, and to keep the circuit open at all times when it is not essential that it should be closed. Otherwise the heating caused by the current may seriously impair the insulation of the coils, and also change their resistance. In the present experiment, and always when a precision exceeding one per cent is desired, the temperatures of the coils and of the wires measured must be taken and proper corrections made as described in the notes on "Temperature Corrections," p. 73.

Measure the quantities necessary for finding the specific resistance at 0°C . of all the samples of wire provided, in terms of both the centimetre cube and the metre-gramme. Compute these, and also for the copper wire, its specific conductivity (Jenkin, p. 252), and its percentage conductivity referred to pure annealed copper as given by Matthiessen.

In the measurement of the specific resistance, to obtain the length of the wire, stretch it between two nails placed in the floor at as great a distance apart as the wire will permit. Mark a point near each end, and measure the length between these points. Insert the wire in the bridge in such a manner as to measure the resistance between the same points. To obtain the temperature of the wire as nearly as possible during the resistance measurement, coil the wire upon the paraffined board or other arrangement provided. Place the thermometer so that its bulb shall be against the wires. Read the temperature, which is to be corrected for instrumental errors of the thermometer, while the resistance measurements are being made. Read also immediately after the measurements the temperature of the bridge coils, inserting the thermometer into the hole provided for this purpose in the vulcanite top.

The weight of the wire is obtained by weighing the whole wire and making proportionate allowance for the end pieces, whose length must therefore be measured. The diameter of the wire must be carefully measured at twenty or more points along its length with the micrometer gauge, correcting for zero error. The mean may be used, although as the resistance varies inversely as the square of the diameter, the use of the mean square would be more exact, and

it should be employed if the differences in diameter are considerable. Avoid bending the wire sharply or winding it into a small coil. Aim at precision of 0.1 per cent in the final result.

Compute from the results obtained for copper the error in each of the component measurements made, which would produce an error of 0.1 per cent in the final result, and give this calculation in the record.

COEFFICIENT OF TORSION.

Any suspending fibre when twisted must exert a torsional resistance. In an instrument of a definite precision, either this resistance must be proved negligible, or must be measured and corrected for. One method is here given of the determination of the torsion correction in a suspended galvanometer needle.

Set up the instrument with as little initial torsion as possible. Read both ends of the index in their zero position. By means of a magnet or otherwise, with as little strain upon the suspension as possible, and without disturbing the position of the instrument as a whole, give the needle one complete rotation. Read both ends again. The difference from zero reading will give the angle α_1 through which the torsion deflects the needle from its initial position. Rotate the needle back to its initial position; read again; twist one rotation in the opposite direction, and read. This will give α_2 . Make a similar series of readings with two rotations in each direction, and then with three. Compute from these a mean value α° of the deflection for one rotation of the needle. The silk fibre is somewhat viscous, and the effect of this in a gradual yielding of the fibre is perceptible when the angle α is large or when the fibre has been many times twisted. This renders the correction uncertain in some cases. Note any effects of this sort that are observable when 10 rotations are used.

The angle through which the fibre must be twisted to produce α is $360^\circ - \alpha^\circ$. The couple which held this torsional force in equilibrium was that produced by the action of the horizontal component H , of the intensity of the earth's magnetic field on the poles of the suspended needle, and this is

$$Hml \sin \alpha,$$

where m = strength of pole of needle, and l = distance apart of the poles of the needle, and this quantity is, therefore, equal to the moment of torsion in the fibre. By laws of torsion, the moment of torsion due to a twist of the fibre through 1° would therefore be $Hml \frac{\sin \alpha}{360 - \alpha}$, and for any twist of ϕ° it would be ϕ times this quantity. This fraction, $\frac{\sin \alpha}{360 - \alpha}$, is called the "Coefficient or Constant of Torsion," and will be here denoted by θ .

The application of θ in the use of galvanometers will be illustrated by the case of a standard tangent galvanometer. (See notes on "Tangent Galvanometer," and "Law and Factor of Galvanometer.") For that instrument, the equation of equilibrium between the force due to a current passing through the coils, the earth's magnetism, and the torsion of the fibre, calling C the current in cgs. absolute units,

$$\begin{aligned} C \frac{2\pi n}{r} ml \cos \phi &= Hml \left(\sin \phi + \phi \cdot \frac{\sin \alpha}{360 - \alpha} \right) \\ &= Hml (\sin \phi + \phi \cdot \theta). \end{aligned}$$

Hence
$$C = \frac{Hr}{2\pi n} \cdot \frac{\sin \phi + \phi \cdot \theta}{\cos \phi} = \frac{Hr}{2\pi n} \left(\tan \phi + \frac{\phi \cdot \theta}{\cos \phi} \right);$$

$$\therefore C = \frac{Hr}{2\pi n} \cdot \tan \phi \left(1 + \frac{\phi \cdot \theta}{\sin \phi} \right),$$

or, in general, for any tangent galvanometer

$$C = K \tan \phi \left(1 + \frac{\phi \cdot \theta}{\sin \phi} \right).$$

For small deflections ϕ , expressed in circular or radian measure, $= \sin \phi$, approx., so that the parenthesis becomes simplified to $(1 + \theta')$, where $\theta' = \frac{\sin \alpha}{2\pi - \alpha}$, the value of α being also in circular measure. This simplification often serves also where θ is so small as to be nearly vanishing. Note that to reduce θ , the suspension may be either lengthened, or made finer, or the magnetic moment of the needle may be increased. To determine whether or not θ need be used in a particular instrument, consider that if it is desired to introduce into C from neglect of torsion, no error exceeding, *e.g.*, 1 in 1000, then as the term $\left(1 + \frac{\phi \cdot \theta}{\sin \phi} \right)$ enters as a direct factor in the computation of C , it must not exceed in amount (1 ± 0.001) .

Compute the percentage error which would be introduced into C from neglect of the torsion correction for the fibre and needle used in this experiment, if they were in a tangent galvanometer, and the deflection was $\phi = 60^\circ$.

LAW AND FACTOR OF GALVANOMETER.

The student must be familiar with the demonstrations regarding the tangent galvanometer, the unit of current, etc. (Notes on "Coefficient of Torsion," p. 75, Jenkin, viii. § 2; Gray, "Abs. Meas. in Elec." Chaps. iii. and iv., etc.)

Form of Standard Galvanometer.—A simple and convenient form of standard tangent galvanometer for measurements to 0.2 per cent or thereabouts, of currents from 0.01 up to 10 or 20 amperes, may be made by the suspension of a magnet or bundle of magnets of about 1 cm. in length, at the centre of a vertical circular coil of one or more turns of wire, the radius of the coil being from two to ten decimeters, and its cross-section small in proportion to its radius. If the coil consists of several turns, a sufficient length of wire should be measured before winding, then carefully and regularly wound into a proper channel, and measurement taken of any portion of this length removed during construction. The resistance should be measured before and after winding, and a proper use of paraffin should insure freedom from defective insulation. If the length is not thus known, it must be determined by sufficiently careful measurement of the diameter or circumference of the coils during or after winding, and a correct count of the number of turns in each layer of the coil. Several details of construction are given in Gray, Chap. iv.

Galvanometer Constant and Factor.—When the dimensions of the coils and magnetic needle, and the torsion of the suspending fibre, are of proper magnitudes, the current in amperes passing through the coils of such a galvanometer may be computed by the formula,

$$C = \frac{10 Hr^2}{l} \cdot \tan \phi = \frac{10 Hr}{2\pi n} \cdot \tan \phi,$$

where H = horizontal component of the intensity of the earth's magnetism,
 l = total length of wire in the coils,
 r = mean radius of the coils,
 n = total number of turns of wire in the coils,
 ϕ = observed mean angle of deflection of needle.

In this expression it is known that $\frac{l}{r^2}$, or $\frac{2\pi n}{r}$, gives the intensity of field produced at the centre of the coil by a unit current passing through the coil. This intensity is of course a constant for any individual galvanometer of whatever form. and in standard galvanometers with circular coils the quantity $\frac{l}{r^2}$, or $\frac{2\pi n}{r}$, may conveniently be denoted by the term "Galvanometer Constant." Following Maxwell's nomenclature, this will be represented by G . Thus, the above formula becomes

$$C = \frac{10 H}{G} \cdot \tan \phi.$$

It will be seen that the sensitiveness of the galvanometer is proportional to G . The galvanometer constant can be computed from the coil dimensions in a few simple forms of coil only, of which that already described is most frequently convenient. In the last preceding expression it will be seen that the whole fraction $\frac{10 H}{G}$, i.e., $\frac{10 H r}{2 \pi n}$, is constant for each instrument so long as H remains the same, a condition ordinarily nearly fulfilled when the galvanometer remains fixed in position, and no disturbance by masses of iron, magnets, or electric currents takes place in its vicinity. Thus, in the ordinary use of a galvanometer, this quantity is a constant factor, by which it is necessary to multiply some function of the observed angular deflection (*e.g.*, the tangent) in order to determine what current (in amperes) produced the given deflection. Thus in

$$C = \frac{10 H}{G} \tan \phi = K \cdot \tan \phi,$$

K may be called the "factor" of the galvanometer in question. If G can be computed, and H is known, of course K can also be computed, and this is the case with the standard tangent galvanometer. For this instrument, the following expression serves to give results to 1 part in 500, provided that the various quantities are determined with sufficient accuracy, that the dimensions of the cross-section of the coil (supposed rectangular) are small (say not more than $\frac{1}{5}$ to $\frac{1}{10}$) compared with the radius r , and that the construction and adjustments are correct:

$$C = \frac{10 H r}{2 \pi n} \cdot \tan \phi \left(1 + \frac{1}{2} \cdot \frac{b^2}{r^2} - \frac{1}{3} \cdot \frac{d^2}{r^2} \right) \left(1 - \frac{3}{4} \cdot \frac{l^2}{r^2} + \frac{15}{4} \cdot \frac{l^2}{r^2} \cdot \sin^2 \phi \right) \left(1 + \frac{\phi \cdot \theta}{\sin \phi} \right).$$

The parentheses give respectively:

I. Correction for dimensions of sections of coils:

$2 b$ = breadth of section (parallel to axis of coil),

$2 d$ = depth of section (parallel to plane of coil).

II. Correction for length of needle:

$2 l$ = length of needle between magnetic poles.

III. Correction for torsion of fibre:

θ = coefficient of torsion, see p. 75.

Any or all of these corrections may become negligible by proper proportioning of parts, and the above expressions are used for computing the proportions.

Find the coefficient of torsion, and compute and record for the standard galvanometer used (note its number), the numerical value of each correction, for deflections of 30° , 45° , and 60° , and apply (by interpolation) in every case throughout this experiment those whose omission would introduce an error of 1 in 1000. The proportions between b , d , r , l , etc., may be computed for any given value of r , by equating the terms $\frac{1}{2} \frac{b^2}{r^2} - \frac{1}{3} \frac{d^2}{r^2}$; $-\frac{3}{4} \cdot \frac{l^2}{r^2} + \frac{15}{4} \cdot \frac{l^2}{r^2} \sin^2 \phi$; and $\frac{\phi \cdot \theta}{\sin \phi}$, successively to the limiting fractional error, *e.g.*, 0.001, and solving for $\frac{b}{r}$, $\frac{b}{d}$, $\frac{l}{r}$, θ , etc. The first will give also value of $\frac{b}{d}$ to make error zero, by equating to zero, and solving for $\frac{b}{d}$. (Maxwell, "El. and Mag." ii, 317; Kohlrausch, "Phys. Meas.," 154 to 157.)

The index should be secured rigidly to the needle, and parallel to its axis. An index at right angles to the needle is necessary in a cosine galvanometer on account of dip, but is undesirable in a tangent galvanometer, on account of the error introduced where the index is not exactly at right angles. With a circle of 4 to 6 inches' radius divided into degrees, a fine index may be read to 0.1° with an error of less than that amount, so that the mean angle readings described below will have an average deviation of perhaps 0.05° , and are therefore precise to that extent, disregarding undetermined errors of graduation. For very small deflections the mirror and telescope are required. The inertia of the needle should be small, and the directive force of its magnet as great as possible, the needle being best made up of several strips of steel strongly magnetized.

In the tangent galvanometer the angle of greatest precision of current measurement is 45° , where, as may be seen from a table of natural tangents, the error produced in the tangent by 0.1° error in reading is 0.0035, and as $\tan 45^\circ = 1$, this is 0.35 per cent. So that the precision of the mean reading at 45° would be about 0.17 per cent by the previous assumption. The precision is the same at equal distances on each side of 45° . Errors of 0.05° at any points between 30° and 60° make less than 0.2 per cent, and between 20° and 70° less than 0.35 per cent in the tangent, so that as far as the influence of errors of reading is concerned, deflections ranging anywhere from 30° to 60° give about the full precision of which such an instrument is capable.

To Adjust the Galvanometer.—1. Raise needle until it swings clear. Do not twist fibre in so doing.

2. *Level* by foot screws, until the plane of the coils is vertical (within 0.2°), judging by a level. Plane of circle should be originally equally nearly at right angles to plane of coils. Turn about vertical axis until needle is nearly in plane of coils, thus putting this plane approximately into magnetic meridian. All disturbing masses of iron must be removed from vicinity. Then repeat levelling if necessary.

3. *Eccentricity.*—Centre needle approximately in graduated circle, if any adjustment of suspension is provided, by making both ends read nearly the same when undeflected, also when deflected to about 70° by a current. Do not use too strong a current. In reading deflections, place the eye so that the index exactly covers its own image.

Middle of needle should be in centre of middle plane of coils. Also adjust circle so that the line of the zero points is as nearly as possible in the middle plane of the coils.

4. *Meridian Adjustment.*—Turn coils on vertical axis until index is as nearly as possible parallel to line of zero points, thus completing 2. Level again if necessary.

To Read the Galvanometer.—5. Always read both ends of needle to eliminate eccentricity. Call reading α_1 and α_2 .

6. Always reverse current, and read α_3 and α_4 .

7. Then $\phi = \frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4}{4}$ eliminates eccentricity, error in adjustment of zero line, error of setting coils in meridian, error from initial torsion; the three last, however, only approximately, but sufficiently when the error is small. Throughout this experiment it is supposed that the standard galvanometer can be read to 0.1° , and that it is desired to eliminate all other errors separately exceeding 1 in 1000.

Law of Galvanometer and Determination of Factor.—In a galvanometer whose coils are not designed with proper proportions between parts, or in such a shape that the corrections above given can be computed, it may be important to determine what law connects ϕ and C , and if this function is nearly that of $\tan \phi$, to measure the deviation from that function, so that the factor and corrections to obtain C from ϕ may become known. This may be done in many ways, of which the two most general are here given. The first is an application of Ohm's law; the second requires the use of a standard galvanometer (supposed in what follows to be also a tangent galvanometer).

By Ohm's Law.—Note always the numbers or marks of all instruments used, and draw sketch of circuit as actually arranged. Connect in direct circuit a constant battery (which should have large plate surface to reduce polarization changes), a known variable resistance, and the galvanometer to be studied (not the standard in this case, although that instrument might of course be tested in this way). Give the resistance successively such values R_1, R_2 , etc., as will produce deflections of about $15^\circ, 25^\circ, 35^\circ, 45^\circ, 60^\circ, 75^\circ$, or as nearly this frequency and range as possible. Then take the series with the same resistances in reverse order. Average the corresponding mean deflections, and use the resulting angles as ϕ_1, ϕ_2 , etc., in the subsequent work. If there are differences in the corresponding mean angles of the two series, these will be in large measure due to changes in the battery during the experiment. If they exceed 0.5° , the battery must be further inspected, and the series repeated. The battery should be put on short circuit for a few minutes before beginning the observations.

1. If the E.M.F., E , and resistance B , of the battery were known, and also the resistance of the galvanometer G , and connecting wires r , then the current causing ϕ_1 would be $C_1 = \frac{E}{B + R_1 + G + r}$, and C might thus be computed for each value of ϕ . A plot with values of C as ordinates, and ϕ as abscissas, would then serve for interpolation in subsequent use of the galvanometer. This process is, however, not suitable in most cases, since E and B are subject to considerable uncertainty. Also, if the galvanometer followed the law of tangents (tested as follows), the galvanometer factor K could be determined from these values of C and ϕ in a manner to be described later in the case where C is measured by a standard galvanometer. This method may be omitted from the present work.

2. Since the usual test to be made is as to whether the deflections follow some supposed law in regard to the currents, *e.g.*, C being proportional to ϕ , to $\tan \phi$, to $\sin \phi$, or in general to any function $F(\phi)$, the problem is simplified. Let the supposed function be $\tan \phi$, so that the supposed expression is

$$C = K \tan \phi,$$

where K is the factor of the galvanometer. Also, in the above observations,

$$C = \frac{E}{B + R + G + r};$$

combining these gives

$$\frac{E}{B + R + G + r} = K \tan \phi,$$

where, as E and K are constant, $(B + R + G + r)$ is proportional to $\frac{1}{\tan \phi}$ and therefore to $\cot \phi$. If, therefore, a plot were made with values of $(B + R + G + r)$ as abscissas, and of $\cot \phi$ as ordinates, the result would be a straight line only when the galvanometer followed the law of tangents (within errors of observation). But as B is often unknown, and it may be inconvenient to measure G and r , the plot is to be made with the observed values of R as abscissas, and of $\cot \phi$ as ordinates. This line, it will be seen, should be identical with the one just described, except that the axis of $\cot \phi$ (ordinates) is shifted by an amount $(B + G + r)$ to the right. If, therefore, the straight line (if such be obtained) be prolonged to intercept the axis of resistances, the intercept will give $(B + G + r)$. If G is known (Wheatstone's Bridge), $(B + r)$ may be found by subtraction, and hence B , since r is usually negligible. In general, the plot would be made with $\frac{1}{F(\phi)}$ as ordinates. If E were known, K would be given by multiplying the tangent of the angle between the line obtained and the axis of resistances by E . If a Daniell cell is used, assume $E = 1.08$, and compute K . Give demonstration of this method of finding K . When two galvanometers are used, as in the subsequent readings, they must be so far apart that a *test* shows that the strongest current passing through either does not affect the other.

Obtain from the plot the point of greatest divergence from the law of tangents between about 30° and 60° , and state the percentage error at this point in terms of the whole tangent. A more precise estimate may be made of the deviation, by computing for each observed deflection ϕ the product $R \tan \phi$, where R is the total resistance in circuit. This product should be constant, and its percentage variation is readily found. Compute the maximum percentage variation between about 30° and 60° .

By Comparison with Standard.—When a standard galvanometer is available, a better test is the following, which should next be made. Connect in series the galvanometer to be studied, the standard galvanometer, the battery, and the resistance box. The same current will now flow through both galvanometers. If the deflection of one is more than twice as great as that of the other, the greater should be reduced by shunting that galvanometer with such a resistance in the form of a wire or a resistance box (see Jenkin, xvi. § 3, or Kempe, "Handbook of Electrical Testing," Chap. iv.), as will make the deflections about equal. If the shunt is employed, the following directions as to computation must be modified, as will be seen by consulting the above references. Vary the resistance until the readings of the galvanometer to be tested are about 10° , 20° , 30° , 40° , 50° , 60° , 70° , 80° , successively, and read each time both galvanometers. Reversals of currents in both instruments and readings of both ends of needle are always essential. Compute the factor of the standard galvanometer, as directed at page 77, assuming $H = 0.1717$, and compute the current C for each deflection of the standard galvanometer. Plot a curve with mean deflections of the unknown galvanometer as abscissas, and currents as ordinates. This curve would serve for interpolation to give the current corresponding to any deflection of the galvanometer, if no simple law connected the deflection and the current. To test whether the galvanometer follows the

law of tangents, plot a series of points with tangents of its mean deflections as abscissas, and currents as ordinates. If the law of tangents is followed, these points will lie along a straight line (otherwise the line will be curved) two of whose properties are of interest. Call K the factor of the unknown galvanometer, supposing it to follow the law of tangents, then $C = K \tan \theta$, where θ is the deflection produced by the current C . Now, when $\theta = 45^\circ$, $\tan \theta = 1$, and thus $C = K \times 1$. Hence, the ordinate of this line at the abscissa 1 (*i.e.*, corresponding to $\tan 45^\circ$) is the factor of the galvanometer. Also, since $K = \frac{C}{\tan \theta}$, K is also given by the tangent of the angle which the line makes with the horizontal axis. If the work is properly done, these two results will of course agree within the limit of precision attainable on the curve, which should be plotted on a large scale.

When the second galvanometer is already known to follow the law of tangents or other function, a single comparison with the standard may of course suffice to determine its factor.

In this method, as in that based on Ohm's law, a more precise discussion than the graphical may be had by computation. If the tested galvanometer follows the law of the tangents, and θ be its deflection corresponding to ϕ on the standard, then

$$\frac{\tan \theta}{\tan \phi} = \text{a constant.}$$

Compute, therefore, this ratio for each deflection of the series, and from the ratios find the maximum percentage deviation between 30° and 60° , as before.

If the galvanometer tested is one (suppose a tangent) of a possible precision of 1 in 500, *i.e.*, about equal to the best standard tangent and to the possibilities of reasonable certainty in regard to H , the foregoing graphical methods are not sufficiently precise. In such a case, the comparative observations having been made as just directed, let the deflections of the standard be represented by ϕ , of the unknown by θ . Then

$$C = \frac{10 Hr}{2\pi n} \cdot \tan \phi = K \tan \theta.$$

Find the mean value of

$$K = \frac{10 Hr}{2\pi n} \cdot \frac{\tan \phi}{\tan \theta}$$

from two or more pairs of observations (those in which θ and ϕ are symmetrically disposed about the 45° point are preferable). Then compute the corrections

$$\begin{aligned} \frac{10 Hr}{2\pi n} \cdot \tan \phi_1 - K \tan \theta_1 &= d_1, \\ \frac{10 Hr}{2\pi n} \cdot \tan \phi_2 - K \tan \theta_2 &= d_2, \text{ etc., etc.,} \end{aligned}$$

for all the observations, and plot values of θ as abscissas and of d as ordinates, on a large scale. Then, in subsequent use of the tested galvanometer,

$$C = K \tan \theta + d,$$

d being found by interpolation on the curve, at the abscissa θ . This method is capable of much refinement and of several modifications to adapt it to special cases, *e.g.*, to commercial instruments of various forms.

From the definition at page 77, it will be perceived that if K be determined by any of the preceding methods for any galvanometer whatever, and H be known for the time and place, then the "constant" G for that galvanometer can be computed once for all. G is a fixed quantity for that particular instrument

so long as its coils, needles, etc., retain the same relative positions, sizes, etc.; and thus the factor would be known subsequently for the same or any other place if the value of H for that time and place were known.

By Electrolysis.—The determination of the factor K may also be made by electrolysis, as will be described in the notes on the electrolytic determination of H .

Sine Galvanometer.—The standard tangent galvanometer, if provided with a vertical axis of rotation, should now be used as a sine galvanometer (see Kempe, "Handbook of Elec. Testing," chap. iii.; Wigan, "Electrician's Pocket-Book," p. 71), by putting in circuit with it the battery and resistance box. Adjust R until the deflection is $\phi = 30^\circ$ to 40° , and read as a tangent galvanometer. Then turn the coils about the vertical axis, and read as a sine galvanometer, first on one side, and then on the other side, of the zero, with the north end of the needle, then similarly with the south end. Let the mean of these four readings be θ . Then $\tan \phi$ should equal $\sin \theta$. It should be observed that the sine galvanometer as thus constructed affords no means of correcting, for want of exact adjustment of needle to plane of coils, except by reversal of current and entire re-setting of the instrument. It will be found, in general, much less convenient in use than the tangent galvanometer, though possessing some advantages.

E.M.F. AND R. OF BATTERY.

The following method is applicable to the so-called "constant" cells only, as would be at once shown by trial with a Leclanché or other inconstant cell. Even with the more nearly constant cells there is sufficient change in the electromotive force E , and resistance B , of the cell during use with varying external circuits to cause the measurements to be somewhat unsatisfactory. This is not to be ascribed to a fault of the method, but to difficulties inherent in any battery in which a liquid electrolyte is used, the polarization effects in liquids being often very troublesome, and at best only partially avoidable. The smaller the currents used in the measurement, and the less their duration, the less the difficulty from polarization.

Connect in direct circuit a Daniell battery, a tangent galvanometer of known resistance G , and known factor K , and a resistance box, R , by which resistance from 0.1 ohm or 1 ohm up to 100 ohms may be inserted. Make the external resistance $R + G$ as low as possible, without obtaining a deflection greater than 75° on the galvanometer, and read the deflection a_1 , calling R_1 the resistance inserted. Then the current is

$$C_1 = K \tan a_1 = \frac{E}{B + R + G}.$$

Make the resistance of such amount as to about halve the current, and read a_2 . Then

$$C_2 = K \tan a_2 = \frac{E}{B + R_2 + G}.$$

Increase R again, and obtain smaller deflections, a_3 and a_4 , in succession. These four deflections should range from about 75° or less, down to about 15° , or as nearly that as the conditions will allow. It is sometimes desirable to give to the ratio of the successive currents the value 2:1, or some special value to correspond to conditions of some case of practice.

Assuming the E and B to remain constant in any successive pairs of measurement, they may be computed as follows. Compute both in this way from

each successive pair of deflections a_1 and a_2 , and a_2 and a_3 , etc. Follow carefully the directions given in the notes in regard to the adjustment and use of the galvanometer, pages 78 and 79.

The factor K is first to be computed from the dimensions of the coils of the galvanometer (unless otherwise specified on the instrument). From this the current C_1 , C_2 , etc., corresponding to the deflections a_1 , a_2 , etc., are of course found from the expression $C_1 = K \tan a_1$, etc. Then, by Ohm's law, denoting by r_1 , r_2 , etc., the total resistance $R_1 + G + w$, i.e., the sum of the inserted resistance, that of the galvanometer, and that of the connecting wires (usually negligible),

$$E = C_1 B + C_1 r_1. \quad \therefore B = \frac{E - C_1 r_1}{C_1}.$$

$$\text{Also, } E = C_2 B + C_2 r_2. \quad \therefore E = \frac{C_2}{C_1} (E - C_1 r_1) + C_2 r_2. \quad \therefore E = \frac{C_1 C_2 (r_2 - r_1)}{C_1 - C_2}.$$

From this value of E two values of B may be computed by substitution in the two equations $E = CB + Cr$, from which the numerical values of E were deduced. The mean of these two values may be used as a value to correspond to the value of E . But a better way of finding B is this. Equating the first two expressions for E gives

$$C_1(B + r_1) = C_2(B + r_2).$$

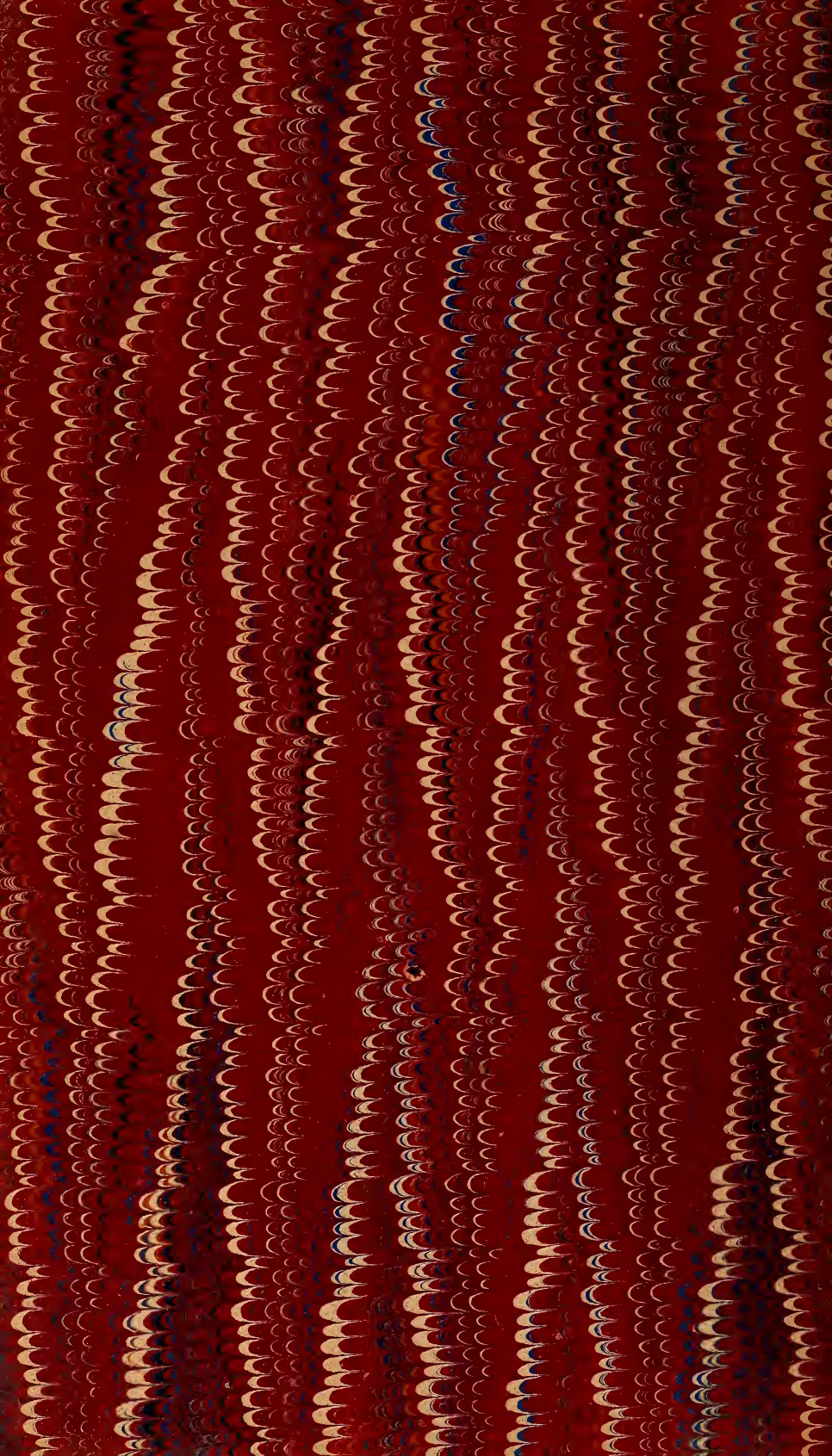
$$\therefore B = \frac{C_2 r_2 - C_1 r_1}{C_1 - C_2},$$

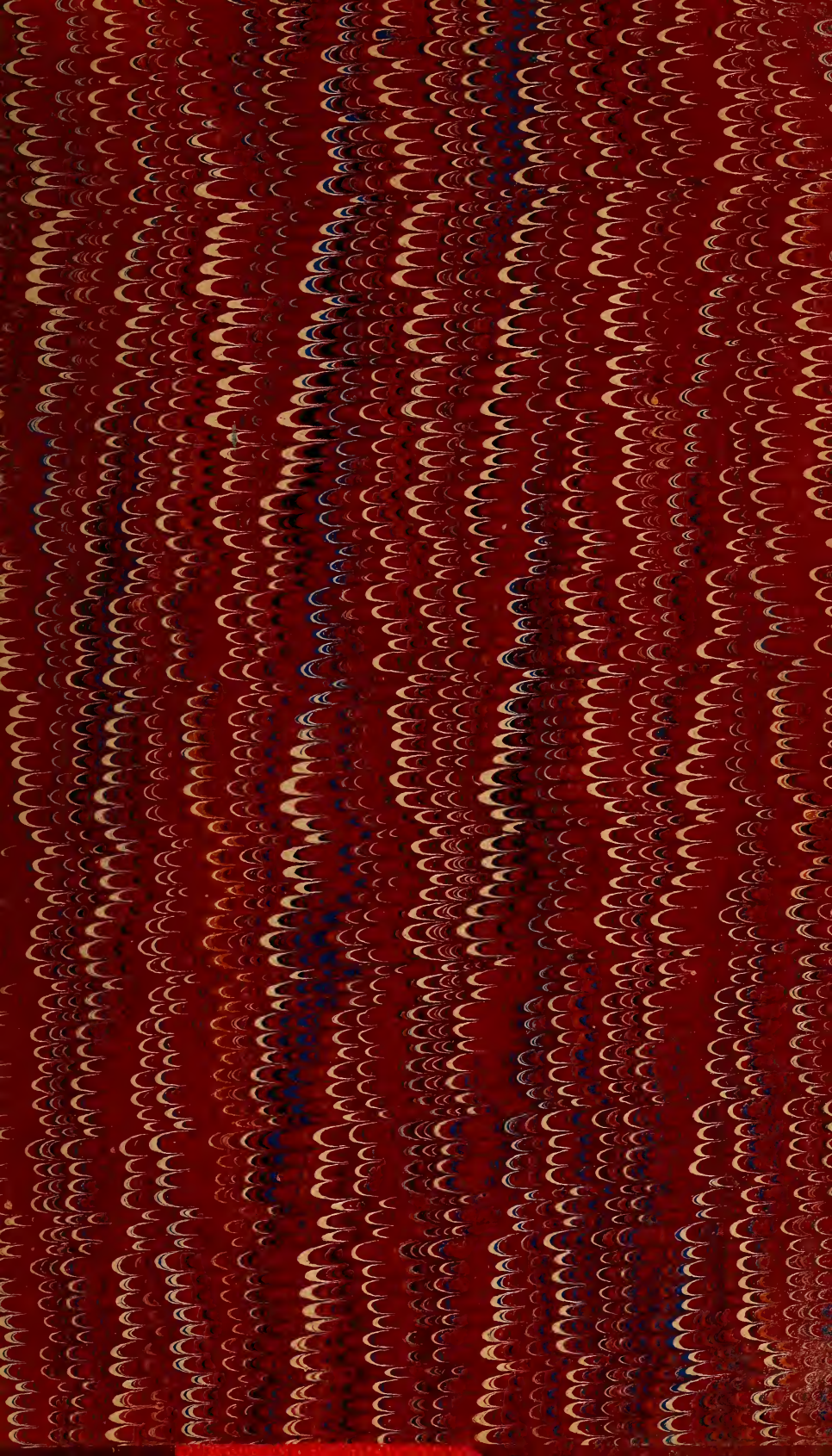
from which values of B may be deduced corresponding to the above values of E . Inspection will show that this contains in both numerator and denominator the common factor K , since $C_1 = K \tan a_1$, etc. Striking out this factor leaves

$$B = \frac{r_2 \tan a_2 - r_1 \tan a_1}{\tan a_1 - \tan a_2},$$

which is independent of K . Thus, the resistance of a battery may be determined by means of a tangent galvanometer (or any galvanometer whose function is known) without a knowledge of K .

The values of $\tan a$ are in all cases to be corrected for torsion of the suspending fibres (see notes on "Coefficient of Torsion," p. 75).





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